Role-playing the Standards for Mathematical Practice: A Professional Development Tool

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Role-Playing the Standards for Mathematical Practice: A Professional Development Tool

Jonathan D. Bostic and Gabriel T. Matney, Bowling Green State University

Abstract

This article describes a performance assessment to use during Common Core-focused professional development and shares insights from research using this assessment regarding teachers’ comprehension of the Standards for Mathematical Practice (SMPs). We asked 46 teachers from grades K-10 to read and make sense of the SMPs and then role-play a classroom scenario indicative of one SMP. This performance task is called Unpacking the SMPs. Teachers’ interactions during the role-play activity were intended to help them interpret the SMPs. From this role-play activity, PD providers were more aware of teachers’ initial comprehension of the SMPs.

Mathematics instruction in the era of Common Core State Standards for Mathematics (CCSSM) will require educators to reevaluate their current instruction (National Council of Teachers of Mathematics [NCTM], 2010). A critical element of the CCSSM is the overarching emphasis given to the Standards for Mathematical Practice (SMPs). The SMPs offer descriptions of mathematical habits and behaviors that students should demonstrate while learning mathematics (Common Core State Standards Initiative [CCSSI], 2010), which are listed in Table 1. The habits and behaviors in the SMPs are just as important as the Standards for Mathematical Content, and are central to developing students as mathematically proficient learners.

Prior research has explored whether and to what degree mathematical habits and behaviors like those found in the SMPs were happening in the classroom. For instance, video analysis of mathematics instruction indicated that generally speaking, teachers were not promoting habits and behaviors like those described in the SMPs (Hiebert et

Table 1: Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Standard for Mathematical Practice #</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Make sense of problems and persevere in solving them.</td>
<td></td>
</tr>
<tr>
<td>2 Reason abstractly and quantitatively.</td>
<td></td>
</tr>
<tr>
<td>3 Construct viable arguments and critique the reasoning of others.</td>
<td></td>
</tr>
<tr>
<td>4 Model with mathematics.</td>
<td></td>
</tr>
<tr>
<td>5 Use appropriate tools strategically.</td>
<td></td>
</tr>
<tr>
<td>6 Attend to precision.</td>
<td></td>
</tr>
<tr>
<td>7 Look for and make use of structure.</td>
<td></td>
</tr>
<tr>
<td>8 Look for regularity in repeated reasoning.</td>
<td></td>
</tr>
</tbody>
</table>

Note: Discussion about a specific SMP is denoted as SMP # within the manuscript.

This manuscript was supported by two grants. Any opinions expressed herein are those of the authors and do not necessarily represent the views of the granting agency.
al., 2005). An implication of this finding is that teachers need opportunities to learn about the SMPs so that they might enact instruction that supports them. Professional Development (PD) providers should design PD that assists mathematics teachers’ understandings of the habits and behaviors found in the SMPs and how they can be promoted through their instruction. Fortunately, teachers want SMP-focused PD during this transitional era from state-level standards to the CCSSM (Bostic & Matney, 2013). Before doing any SMP-focused PD, however, it is prudent to assess teachers’ prior knowledge about the new standards so that the PD best suits their needs. Hence we designed a performance assessment to make sense of teachers’ ideas of the SMP in order to better focus the design of future PD sessions to meet the needs of those teachers.

Performance assessments are just like any other measure: an assessment of and for learning that provide an indicator of an individual’s knowledge (Wiliam, 2007). One type of performance assessment is role-play. Role-play offers a window into an individual or group’s beliefs, thoughts, and actions (Van Ments, 1999; Yardley-Matwieczuk, 1997). It is a performance assessment “in which participants ‘take on’ or ‘act out’ specific ‘roles’ often within a predefined social framework or situational blueprint” (Crookwell, Oxford, & Sanders, 1987, p. 155). If utilized correctly during PD, role-play is a focused and creative enactment of teaching and learning experiences. It places teachers in situations that have the same constraints and pressures that exist in their classrooms (Van Ments, 1999). Role-play has been used previously as a teaching and assessment tool regarding social and affective issues (Jones, 2007; Van Ments, 1999). Our dual purposes in this article are (a) to describe a performance assessment (i.e., role-play) that allows PD providers an opportunity to make sense of teachers’ comprehension of the SMPs and (b) to share what we learned about K-10 mathematics teachers’ comprehension of the SMPs as a result of the assessment.

**Overview of the PD**

Broadly speaking, the aims for the PD included: making sense of the SMPs; exploring inquiry through worthwhile tasks, mathematical discourse, and appropriate learning environments; and implementing classroom-based tasks that aligned with the SMPs and the Standards for Mathematical Content. Each author was a project director for one PD program and co-director on the other. One program supported elementary (i.e., K-5) teachers and the other focused on middle and high school teachers (i.e., grades 5-10). Prior to the PD, none of the teachers indicated that they had read or reflected on the implications of the SMP for their classroom instruction.

There were 46 teachers from a Midwest state who participated in the PD. One project served 23 grades K-5 mathematics teachers while the other supported 23 grades 5-10 mathematics teachers. The K-5 and 5-10 participants met separately due to geographic constraints. Demographic data for the participants are shown in Table 2.

### Table 1: Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Demographic Information</th>
<th>K-5 Cohort</th>
<th>Grades 5-10 Cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (SD) years of teaching experience</td>
<td>11.89 (6.27)</td>
<td>12.97 (5.08)</td>
</tr>
<tr>
<td>Number of female teachers</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>Number of male teachers</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: Demographic information was not available for two participants.

Participants came from urban, suburban, and rural school districts in the Midwest and ranged in classroom experience from one to 26 years. On average, participants across both cohorts had approximately 12 years of teaching experience. Both projects included participants from at least one district with more than 20% of students from families below the poverty line.

Participants met four times for four-and-a-half hour sessions during spring 2012. Our focus in this paper is on the role-play used during the spring dates of the respective PD meetings. The role-playing activity was named Unpacking the SMPs.

**Unpacking the SMPs**

Groups consisting of two to four participants were assembled. Elementary participants were arranged into groups of grades K-2 (i.e., primary) and 3-5 (i.e., intermediate) elementary teachers. Middle level and secondary participants were organized into groups of grades 5-7 (i.e., middle school) and grades 8-10 (i.e., high school) teachers. The grades 8-10 formation was made because many eighth-grade teachers also taught Algebra and/or Geometry. Each group created three role-plays, one for each of three
assigned SMPs: (a) SMP 1 or 6; (b) SMP 2, 3, or 4; (c) SMP 5, 7, or 8. We were careful to assign each SMP to one group within each grade band.

For each assigned SMP, participants carefully and closely read the paragraph descriptions of the SMPs. Following this, each group described the SMP in a manner that the following people might understand: a child in their respective grade levels, a parent or administrator, and a fellow teacher of mathematics. These descriptions were helpful in formatively assessing how participants made sense of their assigned SMPs and intended to share their ideas to various audiences.

After creating the descriptions, participants were expected to role-play three classroom scenarios, each role-play depicting an assigned SMP. Those not participating in the role-play (onlookers) were expected to look for evidence related to that specific SMP. Onlookers did not usually participate in the role-play except on one occasion. Groups were encouraged to behave as the teacher and students or role-play a scenario with only students. Participants were encouraged to choose their mathematical focus, problem, and context, drawing on their typical mathematics instructional experiences to portray their assigned SMP.

For instance, participants could retrieve tasks from websites, computers, or textbooks, and structure themselves to show whole-class, small-group, independent work, or some combination of these formats. There was a ten-minute limit placed on role-plays; however, participants could provide interludes between sequences (e.g., role-play an introductory task, provide some narration, and then segue to a focal problem). Initially, groups were given 40 minutes to prepare their first two role-plays. As participants gained experience creating role-plays, they decided that less time was sufficient. Designing and preparing role-plays took approximately two hours.

Finally, a time of discussion, feedback, and questions over the particular SMP involving all participants occurred after both grade-bands presented their role-plays. The rest of the participants were tasked to share whether and/or to what degree the SMP was evident in the role-play as well as any additional thoughts. After each grade-band shared their role-plays and the group’s discussion waned, the PD leaders synthesized teachers’ ideas. Approximately five hours of the Spring PD were directly devoted to the Unpacking the SMPs role-play task.

Our role as PD providers was to help participants make sense of the objective and to facilitate ways to demonstrate evidence of the SMP. This was accomplished in various ways. A central reason for us to engage participants in this role-play activity was to formatively assess our participants’ understanding of the SMPs. For this initial PD activity, we did not re-direct participants but instead encouraged them to re-read the SMPs and to unpack them according to their own predilection. Our role was to help participants make sense of the language in the SMPs. We encouraged reading strategies and using available resources (e.g., dictionaries) with unusual terms (e.g., decontextualize). Also, we supported participants’ role-play ideas by discussing recent tasks they did in their classrooms that they believed addressed their SMPs. Finally, we initiated and facilitated discussions with questions such as “What did you notice in the role-play?” and “What evidence from the description shared by this group did you see in the role-play?” We also welcomed and encouraged onlookers to ask questions to those presenting the role-play. These questions and the ensuing discussions deepened our understanding of participants’ impressions of the SMPs and provided a rich context for everyone to make sense of the SMPs for classroom contexts.

What Did We Learn from the Role-Play?

We videotaped the activity and examined the visual and audio evidence of the interactions, cues, writing, technology, and expressions used during the role-play and ensuing conversations using narrative analysis (Hatch, 2002). First, videos of the unpacking activity were transcribed. A table was created to organize ideas during the coding process. Each SMP was ascribed a column and each group of teachers was assigned a row. Second, we watched the videotapes and read transcripts simultaneously to familiarize ourselves with the data. Videotapes were paused after each role-play to allow the coders to discuss the activity. Initial ideas about each group’s role-play were recorded as memos to reflect on during iterative and subsequent analyses. Next, we reviewed the memos within the matrix for overarching impressions that transcended across groups, grade levels, and/or SMPs. Later, impressions were reexamined for substantial evidence and a paucity of evidence. Impressions were retained when there was substantial evidence from the videotapes and/or transcripts. The final stage in the process was to rewrite the impressions as complete thoughts. The following findings are impressions...
of our participants’ initial comprehension, which might be shared by teachers in your state, county/district, or city. We noted the teacher in the provided excerpts from the role-plays; all others in the role-play behaved as students. All names are pseudonyms of participants who enacted role-plays.

**Impression #1: SMPs Are For Students**

The first impression was that participants struggled with the notion that the SMPs are written for students to demonstrate. This is clearly evident in the language of the SMPs because every standard begins with “mathematically proficient students” (CCSSI, 2010, pp. 6-8). Ideally, the teacher should act as a facilitator, creating a context for students to exhibit these mathematical behaviors. The videos of the role-play consistently showed that participants struggled with determining what it is students should exhibit as evidence of the behaviors in the SMPs. For example, the teacher in the grades 5-7 role-play for SMP #7 did not allow students to wrestle with a mathematics question. The teacher (Angela) in the 90-second role-play showed that she was able to demonstrate evidence of this SMP but her students (Ryan and Benjamin) did not have any such opportunity.

*Angela (Teacher):* Okay class, I am going to give you a story problem and I want you to figure it out in your head without using your calculator. We have 15 students in our classroom, and I want to give each student 9 M & M’s. And I need to know how many M & M’s I need to bring to school? 15 students are going to get 9 M & M’s each.

*Ryan:* I don’t know what 15 times 9 is.

*Angela:* Well how can you figure this out? . . . Is there something in there that you do know? Look at your numbers. You should break 15 apart, maybe.

*Ryan:* 15 is 10 plus 5, it is. And I know that 10 times 9 is 90.

*Benjamin:* And 10 times 5 is 45. Oh sorry, I mean 5 times 9 is 45.

*Angela:* OK. So what could you do with 90 and 45?

*Ryan:* Well, we could add those together, and then we get a 135.

In this role-play, the teacher led the instruction using an initiate-responder-evaluate (IRE) format and directed students’ thinking with guiding questions. IRE is a teacher-led three-turn sequence that involves a teacher question, a students’ response, and the teacher’s evaluation of the student’s response (Durkin, 1978-1979). Angela could have used wait time or posed an easier but similar question when students were struggling with large two-digit numbers. The only one providing evidence of looking to make use of structure to solve the problem is the teacher, who quickly offered the idea that 15 could be decomposed into 10 and 5 and then a few seconds later that something should be done with the two partial products (i.e., 90 and 45). The students used the provided hint of structure, but did not look for it themselves.

Students in the role-play were not provided with an appropriately rich problematic task, much less time to wrestle with it, and were not expected to demonstrate the behaviors indicated in SMP #7. Keep in mind that these role-plays were developed and practiced by the entire group, not just the teacher (Angela). The voice of who provides evidence for enactment of the SMP becomes clear through the role-play: the teacher. This episode was consistent with the other role-plays as those who played the role of the teacher demonstrated mathematical habits and/or behaviors described by the SMPs and perceived their role as model for students. Those playing the role of the students tended not to demonstrate habits and/or behaviors. Thus, participants felt that the teacher’s role was to demonstrate the behaviors and habits described by the SMPs and encouraged students to notice how the teacher behaved mathematically.

**Impression #2: Classroom Norms Impact Students’ Outcomes**

The second impression was that the norms of classroom environments impact the depth and quality of the SMP that may be exhibited, and participants seemed unaware of their influence. Expectations for learning, doing, and justifying mathematics are called sociomathematical norms (Yackel & Cobb, 1996). All but one role-play demonstrated the same two sociomathematical norms: students should respond only to teacher questions; and students should not engage in collaborative mathematical thinking. Teachers in these role-plays used an IRE discourse pattern. Additionally, we noticed that one group’s role-play demonstrated participants’ awareness of effective classroom
norms and expectations that fostered SMP-like behaviors.

This group of intermediate elementary (i.e., grades 3-5) participants was asked to role-play SMP #4: Model with Mathematics, although they also showed evidence of other SMPs. The task within their eight-minute role-play was for students to design a field trip that kept students in the local community, maintained low fuel costs, and took students to interesting places. The teacher, Sandra, provided clear directions and then asked the students, Bart and Sarah, to begin working. Sandra started the role-play with the following task:

You guys are going to get to plan our field trip, but we do have some guidelines that you have to follow. The first is that each group will come up with a plan and an idea, an itinerary of our day, of possible places that we can go in our community... each group will have to pitch their idea to the class and then we'll vote on it and whichever one wins that's the field trip we'll get to go on... here are our limitations:... we don't have a lot of money and gas is expensive; so we only get to take the bus 25 miles... The other thing is we are going to leave from school, but we have to be sure that we get back here... You've got to explain to us where we are going to go and about how many miles it is going to take because we got to make sure we get back to the school.

The two students in this role-play, Bart and Sarah, created diagrams (i.e., models) characterizing their proposed field trip and shared them. Sandra asked Bart and Sarah to share interesting mathematical elements within their models (e.g., the order in which places on the field trip route are attended did not affect the overall total number of miles traveled). Bart and Sandra later critiqued each other’s models and responded to questions from Sandra (i.e., SMP #3). Finally, Bart and Sarah showed that they were able to decontextualize the mathematics from a local area map, apply mathematics procedures to develop their models, and contextualize their findings within the field trip problem (i.e., SMP #2). Specific to this role-play, the teachers enacted norms such as students are expected to (a) discuss the effectiveness of the model and its representation, (b) discuss the mathematics within the model, and (c) reason quantitatively as described in the second SMP. This example characterized how a rich task, as well as mathematical and sociomathematical norms, influence students’ engagement in the SMPs.

Impression #3: Misunderstanding SMP #1

The third impression was that there is a lack of evidence that our K-10 participants sufficiently understood the language within SMP #1. Participants’ role-plays provided little evidence of any behavior described in this standard. For example, the high school participants role-played a scenario in which students worked with a system of equations using a graphing calculator. Language within SMP #1 stated that “older students might, depending on the context of the problem...change the viewing window on their graphing calculator to get the information they need” (NGAC, CCSSO, 2010, p.6). This role-play lasted approximately two minutes.

Harper (Teacher): Class are you ready? We talked about how a system of equations with only one solution as a single ordered pair that works in both equations and only those two numbers work. I’d like to give you a new method of finding that solution by graphing the equations. Do you have your graphing calculators? Are they turned on?

Quinn: My batteries are dead.

Harper: Your problem is going to be to graph a system of equations. I would like you to graph \( y = -\frac{1}{3}x + 22 \) and \( y = 2x - 20 \) on your graphing calculator then try to find the solution graphically to that system.

Quinn: Only got one line on here.

Harper: Did you graph the other equation?

Quinn: Yeah I typed them both in here. I got one line.

Harper: Why do you think that is?

Quinn: You use bad parameters?

Harper: Why don’t you think about why you can’t see that solution? Do you think there’s another line in there?

Quinn: I can’t see anything.

Harper: I want you to work as partners and try to solve that problem and reach a solution. (Pause) ... You see two lines but can you give me the solution for it? ... Can you give me the numbers from that picture you’re looking at on your graphing calculator?

Quinn: 0.5 and 8.

Harper: That’s very good.
These participants interpreted expanding the graphing window to examine a system of equations as evidence of making sense of a problem and persevering in solving it (i.e., SMP #1). A critical component to demonstrating SMP #1 is providing students with a worthwhile task that is problematic. No role-play for SMP #1 provided evidence of a problem or rich task that might engage students in perseverance or sensemaking about mathematical concepts or procedures. Furthermore, the intermediate elementary group discussed earlier in impression two was the only group to employ a problem. Note that here we define a problem as having three characteristics: a solution is not obvious; it is uncertain whether a solution exists; and a solution strategy is not readily apparent (Schoenfeld, 2011).

**Summary**

Participants attending these voluntary PD were motivated to improve their teaching and the role-play provided an assessment of their initial comprehension as well as a tool for fostering their learning about the SMPs. These impressions provided insight into teachers’ comprehension of the SMPs and also pointed to features that we considered when enacting our PD that focused on the SMPs.

**Implications for CCSSM-focused PD**

An important implication stems from our first impression. Participants’ role-plays suggested that participants thought they should demonstrate mathematical habits and behaviors described in the SMPs so that students might take them up. It was rare when these participants created a role-play where students engaged in the SMPs. After the role-plays, we discussed that the CCSSM were written for teachers and students. Participants commented that they were aware of this but did not demonstrate their awareness through the role-plays. There was a consensus among participants in both PD programs during these initial meetings that they felt their role as the mathematics teacher was to demonstrate habits and behaviors of mathematically proficient citizens; yet they tended to focus on how to carry out a set of procedures to solve a problem.

The teacher’s role in helping students enact the SMP in their own learning must move beyond simply being a model and hoping that students will pick up on it. We would not have known the pervasive difficulty of this idea among our participants had we not assessed our participants’ comprehension in a way that connected their ideas with classroom practices. Teachers may need support thinking about ways to gather evidence of students’ engaged in the SMPs. Thus, we advise that mathematics education leaders develop PD tasks that remind teachers of the target audience for the CCSSM.

Another key implication of this study comes from the second impression. The role-play assessment was a useful tool for garnering shared experiences among PD participants to highlight important components of good teaching. We were able to assess that most participants paid relatively little attention to the way sociomathematical norms influence student’s enactment of the SMPs through the role-play activity. Although that revelation was important for us as PD providers to assess, even more important to the PD was that there was one group’s role-play that was an exception to the others. From this exception emerged an opportunity of shared experience among our participants to discuss the importance of sociomathematical norms through the lens of role-play done by Sandra, Bart, and Sarah. The role-play was not only a performance assessment by which we could come to understand the sense our participants were making of the SMP but also a task through which overcoming difficulties in teaching and learning could be explored.

From the third impression we learned that it is important for leaders in mathematics education to be careful about our assumptions regarding the sense teachers are making of the SMPs. We have been providing CCSSM-focused PD for teachers since shortly after the document was launched. Most teachers who attend our PDs explain that they have read the titles of the SMPs but never the paragraph below each title. Impression three highlights the difficulty some teachers have in making sense of a particular SMP even after a careful and close reading of the paragraph and discussion of that SMP with other teachers. The role-plays gave all of us, participants and PD leaders, a space through which we could discuss the meanings of the SMPs individually and as a group in more depth and explore habits of mind and behaviors of mathematically proficient students.

As we look to the future, we plan to refine Unpacking the SMP. First, we plan to restructure the description in such a way that teachers might paraphrase the SMPs’ descriptions. While the three unique descriptions were useful, participants struggled to sense the difference between an explanation for a principal/administrator and parent/guardian. Second, we will ask teachers to construct the paraphrased
description and share it during one PD meeting, gather potential materials for the role-play afterwards, and then plan and execute the role-plays at the following meetings.

**Conclusion**

The dual aims of this manuscript were to discuss a performance assessment (i.e., *Unpacking the SMPs*) that allows PD providers an opportunity to make sense of teachers’ comprehension of the SMPs and share what we learned about our participants’ comprehension of the SMPs. The role-play activity was a useful performance assessment because it allowed us an opportunity to formatively assess participants’ prior knowledge and initial comprehension of the SMPs. Our results from the role-play provided information regarding participants’ ideas about the SMPs. An important benefit of the role-playing task was the rich data it provided about how the participants were making sense of the SMPs. The *Unpacking the SMPs* task offers clear benefit for any mathematics education leader aiming to support teachers’ sensemaking of the SMPs.
References


