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Metacognition in a Mathematics Classroom

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I. Abstract: *The purpose of this action research study is to explore the connections between students' ability to engage in metacognitive methods, their understanding of mathematical content, and their mathematical performance. By having a group of students engage in a lesson about metacognition and a mathematical modeling problem then comparing their test scores to that of a control group a correlation can be found to analyze the effects of metacognition methods in a mathematics classroom.*

II. Introduction:

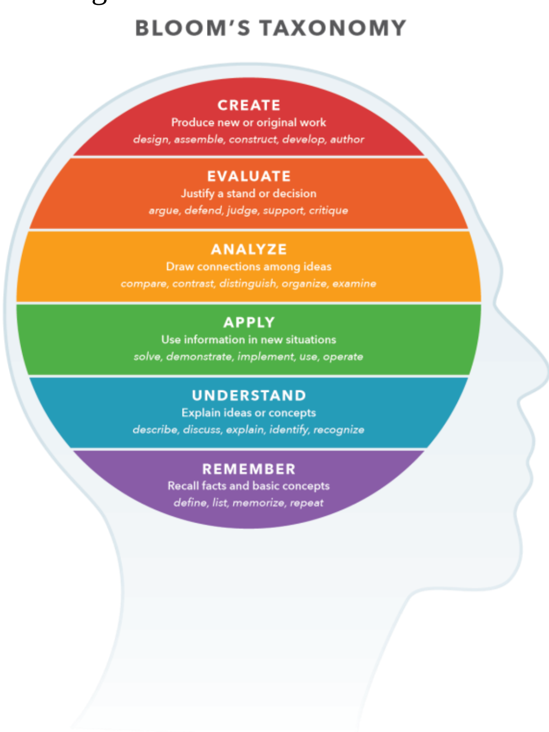
Many mathematics teachers struggle to find ways to engage students in higher-level thinking about mathematics. It can be difficult for students to push past the barrier of “plug and chug” formulas, rules, and steps that often are characteristic of a secondary mathematics classroom and cross into the domain of mathematical discourse. True understanding of mathematics comes not from the memorization of formulas, steps, or rules but from discovering the concepts, ideas, and interconnected parts of mathematics in your own way.

Through my time as a BGSU mathematics education student, I have explored many of the mathematical concepts I will be teaching by articulating my thought processes along the way. It could be argued that because my coursework required me to explore these topics with depth and reflection, I am better prepared to teach them to my future students. I believe the most important part of my education at BGSU is not the content that I am learning, but the way my professors encourage me and my classmates to articulate our thought processes and explain them to our peers. This idea of thinking about one’s thought process is known as metacognition. This process is imperative to math education because it exposes students to deeper understanding in mathematics. Students move beyond, “what’s the formula, rule, or steps that I have to follow to solve this problem?”. It forces them to consider alternatives and think through why they chose to solve the problem in their unique way beyond, “that’s the way my teacher/book/peer told me to do it”. With this idea in mind, I intend to perform a research study on how exposing and engaging students in metacognition improves their mathematical performance and reasoning skills.

III. Literature Review:

When looking at past research on metacognition I found and examined a clear definition of what metacognition is so that I could best conduct my research. According to Ader (2019), metacognition was defined by Flavell (1979), one of the first researchers to use it, as, "knowledge and cognitions about cognitive phenomena (p. 206)" (pp. 615). While this definition is short and concise, it doesn't clearly define the term. A better definition that is referenced in *Teach yourself how to learn: strategies you can use to ace any course at any level* by McGuire, is, "'… one's knowledge concerning one's own cognitive processes and products, or anything related to them, e.g., the learning-relevant properties of information or data' (Flavell, 1976, p.232)" (pp. 22). This definition helps to more clearly connect metacognition to the context of the research I am investigating with the objectives of knowledge and learning. There are many other terms that have been created in relation to the global concept of metacognition, including self-regulation. Adler (2019) as well as Veenman (2006) describe these terms as along-side one another in the learning process. Metacognition is the knowledge and understanding portion while self-regulation fuels the motivation and emotion portion (pp. 614) (pp. 4). Overall, metacognition is simply a person's knowledge of their own thought processes, the final result, and any other things connected to their thought process. After first understanding what metacognition is, now we can discuss how it relates to education and learning more specifically.

Metacognition in the classroom is a student's understanding of the process they completed independently to arrive at a solution to the problem. A student's thoughts about their problem-solving processes and being able to explain those thoughts is important to determining their understanding of the content. In a National Council for Teachers of Mathematics article published in 1985, authors Garofalo and Lester describe the more regulatory aspect of metacognition as focusing on the multitude of decisions and calculated activities that a student might go through during class while working on a cognitively engaging task or problem (pp.166). This idea further connects self-regulation and metacognition in the process of students' learning in a classroom. The book *How People Learn: Brain, Mind, Experience, and School* edited by John D. Bransford describes metacognition as an important part of a student's learning progression. The editor describes that prior knowledge and understanding one's own strengths and weaknesses in the learning process; as well as aspects of self-regulation such as a student's ability to organize their own learning and plan for the successes and failures that will come and learn from them all together are necessary for purposeful student learning (pp. 97). This idea of metacognition and self-regulation as a necessary part of student learning is also highlighted by Paris and Winograd. The authors describe the importance of metacognition as allowing students to understand their own personal thinking and work to be independent learners (pp. 7). Paris and Winograd also describe "consciousness raising" in relation to metacognition and the benefits of this strategy. Benefits include teachers sharing the responsibility for organizing and planning the learning with their students as well as encouraging positive opinions in relation to learning and motivation in students (pp. 7). This further demonstrates the positive effects in learning that occur when engaging students in metacognition and self-regulation skills. Overall, a student's ability to engage in the cognitive skills of metacognition and self-regulation are closely related to their ability to learn and understand content.



Metacognition is connected to many aspects of education such as Bloom's Taxonomy and the difference between studying and learning. Student learning is broken into a hierarchy of learning levels by Bloom's Taxonomy. The second and third highest objective levels, evaluating and analyzing, can be directly related to metacognition. Evaluating is described by McGuire (2018) as making decisions based on standards by examination and reviewing something. Analyzing is described as breaking content into its parts and understanding how those parts are related and connected (pp. 31). By definition metacognition is related to the analysis level of Bloom's objectives and skills classification because the process a student goes through involves examining and breaking down their thought process. McGuire goes on to explain the difference between studying and learning based on student responses. Some students' answers that she cited in her book are, "Studying is memorizing for the exam; learning is when I understand it and can apply it. [and] Studying is short term; learning is long term" (pp. 25). Students believed that they understood the content completely after just the lessons their teacher taught, then they only needed to revisit the material right before their test (pp. 25). In reality, when students take the task of explaining the content to their peers, they predict the multiple different perspectives their peers would have, and the questions their peers would ask which helps them realize they didn't completely understand the content in full themselves (pp. 29). This role forced students to recognize the depth of their own content knowledge and learn from the thoughts and questions presented by their peers. This shows how real, deep learning, or truly understanding content in this manner is related to metacognition.

Metacognition can also be connected to the ability to master content and the Common Core State Standards. In order to master content, a student, teacher, or any learner in general must fully comprehend the content. To get to this level of understanding, a learner has to recognize their progress in mastering the content which is done by monitoring one's understanding and approach to solving problems in that content. In *How People Learn: Brain, Mind, Experience, and School*, the author describes the capacity to distinguish if an approach to solve a problem is the best way or not, as the difference between a novice and an expert in a field. Furthermore, this ability is also metacognitive because when a learner looks at their first approach and takes a step back, they can evaluate their own understanding of the content (pp. 50). Overall, when someone is an expert in a content, they can see their content from the perspective of other experts and learners while questioning their own understanding based on these new perspectives. In the book *How People Learn: Brain, Mind, Experience, and School* the author states that, “A ‘metacognitive’ approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them” (pp. 18). It goes on to explain that when students were asked to articulate their thought processes, they examined their own knowledge ensuring they knew where they needed more information for comprehension. If what they learned was supported by prior knowledge and any personally made connections to previous learning, they could deepen their understanding. This is all part of an internal dialogue that models metacognitive processes. This idea of verbalizing thought processes directly relates to the third Standard for Mathematical Practice is (National Governors Association Center for Best Practices), “Construct viable arguments and critique the reasoning of others”. This standard requires students to be prepared to explain their thought process at a deep enough level to successfully defend it to their peers. This standard also requires students to think through how and why their peers came to their own conclusions to the same problems. Additionally, this standard requires students to verbalize their thought processes and the logical reasoning behind them. Overall, this shows that when students engage in SMP 3 they must think through their thought process, which makes a clear connection to metacognition.

Metacognition is related to many other ideas in mathematics education such as problem solving, mathematical reasoning, and self-regulation and many studies have been completed investigating these connections. Ader (2019) describes the process of problem solving as having four phases, which have significant overlap with Zimmerman's self-regulation process. The first phase of creating a plan is related to forethought. Implementation is related to performance and reassessing one's thought process is connected to self-reflection (pp. 616). When a student engages in problem solving strategies, they think through how to solve the problem, attempt to solve it, and if their plan doesn't work revisit and revise it based on the knowledge they have gained from their earlier attempts. Ader (2019) goes on to connect problem solving and self-regulation to metacognition by describing a teacher creating a classroom environment that encourages problem solving which subliminally stimulates students engaging in self-regulation and metacognitive processes. In *Metacognition, Cognitive Monitoring, and Mathematical Performance,* Garofalo states that mathematics teachers and educators are responsible for incorporating metacognitive enrichment into the lessons they give as they train students' minds to apply formulas and procedures specific to their course content. They must also help students embrace a metacognitive view on their own mathematical performance, so they are constantly examining their own knowledge and thought processes in order to improve their understanding (pp. 173). A math teacher's responsibility is to teach their students not only the content of their course but also how to make connections and use reasoning within their thought processes in order to solve mathematical problems. In the article by Montague (2011) researchers studied if the instructional strategies collectively known as "*Solve it!*" improved the mathematical problem-solving skills of students with learning disabilities (pp. 262). The *Solve it!* process directs the students to manipulate a mathematical problem into words that made sense to them, use the words to represent and visualize the problem, attempt to solve the problem, and finally check their work (pp. 263). *Solve it!* has integrated metacognitive and self-regulatory processes such as self-questioning and self-monitoring by having students thinking through their process of solving the problem in order to check it (pp. 263). After engaging a group of students with this process and measuring their ability to mathematically problem solve, the results were compared with a control group which was not exposed to the strategies accompanying *Solve it!*. The findings of the article state that the students who received the intervention made considerably greater growth in their ability to engage in mathematical problem-solving when compared to the control group (pp. 269). This process of engaging students in metacognitive processes and breaking down mathematical problems allowed them to better understand the problem and how to correct their reasoning if there was a mistake. Overall, the processes of mathematical problem solving, self-regulation, and mathematical reasoning are all interconnected and also related to the cognitive processes of metacognition.

There are many studies that demonstrate how the ability of engaging students in the interconnected metacognitive processes improves their mathematical performance. In an article by Kramarski (2003) researchers investigated the effects of four different instructional strategies involving metacognitive processes and self-regulation on students' mathematical reasoning and academic performance (pp. 281). All four groups of students had female teachers with over 5 years of math education experience. They received the same training, the demographics of all four schools were similar in size, and they were all of average socio-economic status, but their teachers used four different instructional strategies (pp. 285). The four instructional strategies were cooperative learning with metacognitive training (COOP + META), individualized learning with metacognitive training (IND + META), cooperative learning without metacognitive training (COOP), and individualized learning without metacognitive training (IND). The groups which engaged in metacognitive strategies used three sets of self-addressed metacognitive questions including comprehensive questions, strategic questions, and connection questions (pp. 286). Each set of questions focused on engaging students in the different metacognitive processes of reflecting over a problem before solving it, considering how to solve the problem, and connecting solving the problem to prior knowledge, respectively (pp. 286). By having students engage in self-questioning, they are forced to stop and think at each step of the problem-solving cycle. This analysis and reflection ensures that the steps they follow to solve the problem make logical sense. The results of the study show that on average the students in the COOP + META group performed considerably better than the students in the IND + META group, and those students significantly outperformed the COOP and IND groups (pp. 295). We can conclude that when students engage in metacognitive training it deepens their content understanding and subsequently increases their performance.

A second study published in 2020 titled “Motivation and Emotions: The Impact on Metacognition Strategies and Academic Performance in Math and English Classes” focused on how metacognitive strategies improve performance of students in mathematics. The study predicted that teacher support of motivational and emotional learning in the classroom influences the use of metacognitive strategies which all together increases academic performance (pp. 3). In this study, the participants were over 500 high-school students and produced similar results compared to the first study discussed. Trigueros describes the connection between motivation and metacognition which together had a positive association with academic performance (pp. 7).

This may conceivably be due to the fact that metacognitive strategies engage students by nurturing self-awareness as they choose, organize, and monitor their thought processes (pp. 7). When students have the motivation to engage in these deep cognitive processes it allows them to deepen their personal understanding of the concepts that they are learning as well as their ability to advance mathematical reasoning and problem-solving skills. Finally, this study also shows that when teachers provide an environment that promotes self-sufficiency, students are more likely to engage in metacognitive processes as well as motivational and emotional learning which can increase their mathematical performance.

IV. Methodology:

To begin my action research, I selected two class periods of Algebra 2 to be my participants. The students in these two class periods have performed similarly to one another on previous content tests and both classes consist of 18 students. I designated the second period class as the control group and the third period class as the experimental group. This research was performed during the instruction and testing of the content in chapter 3 of the students' textbooks focusing on functions, which occurred from November 3rd, 2020 through November 18th, 2020. During this time these two class periods received the same homework, quizzes, and tests, with slight variations in instruction.

The first day of this unit in the third period (experimental group) consisted of a "Think-pair-share" activity, a group discussion-based activity, and the completion of an exit slip. The "Think-pair-share" activity focused on the question, "What is a function?". Students were given individual time, then partner, and finally whole-class discussion time for this question. According to McGuire (2018), when students take the task of explaining the content to their peers, they predict the multiple different perspectives their peers would have, and the questions their peers would ask which helps them realize they didn't completely understand the content in full themselves (pp. 29). This "Think-pair-share" activity engages students in both explaining their thought process to their peers, collaborating with a peer to arrive at a conclusion, and sharing the conclusion with the whole-class. The group discussion-based activity consisted of students deciding if six different mathematical relationships were functions or not. Students discussed and defended their ideas to their group members as well as to me as I walked around and asked students questions about their ideas. Specifically, "How do you know that relationship is a function?" and "What is the input/output for this relationship?". These questions were intended to prepare the groups of students to share and defend their ideas to the whole class in our discussion. This activity engages students in metacognitive discussion of their thought process and defending their thought process. Students then completed an exit slip, Appendix B, where they were again asked to explain why they believe the equation represents a function, thus reinforcing the importance of defending one's thought process. Finally, after completing the exit slip students were instructed to answer the following question, "As we begin a new unit what study strategies have worked for you in the past? What new habits do you want to try or learn to better your study skills?". This was the first step in helping students take control of their own learning by articulating what strategies worked for them in the past and setting a goal for what new strategies they want to learn. When students are given the opportunity to set goals for themselves, they begin the process of self-regulation. Throughout this unit students were asked about what strategies that we have used in class and that they have used at home have been helpful to deepen their understanding of the content. As well as what they are actively doing in-class and at home to achieve their goals and get their questions answered.

The second day of instruction consisted of a hands-on activity where students constructed a foldable tool used to explore domain and range of graphs visually. While students worked to define the domain and range of graphs visually, I walked around and asked students the following question, “What questions are arising for you during the class session?”. The goal of this question was to help the students begin to dive deeper beyond simply completing the task and think through their misunderstandings of the content within their individual thought processes. This is another example of students engaging in metacognitive thought processes and self-regulation to deepen their understanding of the mathematical content.

The third day of instruction involved students using technology to explore the algebraic patterns for connecting the equations to their domain’s graphically. This discovery-based learning engages students in metacognitive questioning strategies to arrive at conclusions/rules for the patterns they see through their thought process. The goal of this lesson was for students to discover the patterns that the denominator of a fraction cannot be zero and the number under a square root cannot be negative. At the conclusion of this lesson students were asked the following question, “Are there any parts of today’s lesson that you had trouble with?”, and they were told to indicate them in their packet. This is yet another opportunity for students to think through their own thought process to identify gaps in their understanding, thus taking more ownership of their learning and self-regulating.

The fourth day of instruction consisted of exploring piecewise functions by cutting apart the whole graphs of each individual function and putting the pieces together to make a new function. Furthermore, the students also practiced graphing the functions by hand after exploring visually. At the conclusion of this lesson students were asked, “What content from this week was the most challenging?”. This provided students an opportunity to reflect on the content from the week and think through what they struggled with individually. By doing so, students were engaging in more self-regulation and metacognitive questioning. The goal of this question was for students to begin to think about what content they needed more practice with before they take their first quiz the next day.

The fifth day of instruction consisted of a quiz over the first four sections of content, Appendix C, and a lesson on the average rate of change. After completing the quiz students were asked to respond to the following questions: “Based on the quiz today what content/concepts from the past few days did you struggle to understand?” and “What study strategies have you used so far this unit, have they proved successful, and if not, what new strategies do you want to try/learn?”. I took both students’ grades on the quiz and their responses into account as I shifted instruction for the rest of the unit. Many of the students struggled with domain and range both visually and algebraically, and many students indicated in their responses that they were struggling to understand piecewise functions, even though they were not covered on the quiz. The students also indicated in their responses that the main study strategy they had used was looking over their notes and homework, but they wanted to learn how to find good practice problems to use to study as a new strategy. Therefore, we took an extra day of instruction to do a practice day.

The sixth day of instruction was a practice day dedicated to domain and range as well as piecewise functions. As a class we walked through practice problems for defining domain and range of graphs, defining the domain based on an equation, graphing piecewise functions, and evaluating piecewise functions. After walking through each type of problem I gave the students multiple practice problems to complete, focusing on each area one at a time. While students worked on the practice problems I walked around and ensured all of the students’ questions were answered. At the conclusion of the class period, I explained to students that the best place to find practice problems was in their textbook. The online textbook the students use provides the answers to all of the odd practice problems, therefore students can attempt problems and check their answers to ensure they are completing them correctly. Another great resource I suggested was Khan Academy, students can search for the content they need to practice and there are practice problems for all of the content they will need. By taking students responses into account and providing them with what they determined they needed the students felt more in control of the learning process and thus more ownership and responsibility for learning the content.

The seventh day of instruction consisted of another quiz, Appendix D, which served as an opportunity for students to not only showcase their knowledge of the content but identify any still existent gaps in their individual understanding. The eighth day of instruction consisted of learning about the key features of a graph, and I continued to monitor students' progress in their study strategies. Students were asked to reflect on their individual study strategies as well as which lessons, they felt they understood more than others in order to deduce how they like to learn. The ninth day of instruction consisted of combining functions with arithmetic operations and students were asked, "What do you need to actively go and do now to get you questions about today's content answered and your confusions clarified?". This question further emphasized the students ability to think through their knowledge and engage in self-regulation to determine their questions and make a plan of how to get them answered. Finally, the last day of new instruction consisted of compositions of functions and students were asked, "How do you plan on preparing for our upcoming test? Why?". The goal of this question was to tie together all of the goal setting and study strategies the students had developed over the course of the unit, their thought process about why they have chosen such strategies, and their ability to defend their choices using knowledge that they have gained about themselves as learners.

At the conclusion of the unit, after a review day, both classes of students were administered the same unit test, Appendix E. The students were tested on all of the content from the chapter and the scores of both classes of students were compared. Furthermore, during the unit both classes were administered the same two quizzes and their scores were compared. These scores and their comparisons serve as the quantitative data. After students had completed the unit test, they were instructed to complete the same Google Form asking them, “How do you personally prefer to learn new mathematical content?” and they chose from the same list of responses. These responses were recorded and compared both pre and post instruction for each class period and the class periods were compared to one another post-instruction. These responses and their comparisons serve as the qualitative data.

V. Data and Analysis

In this section I will discuss the scores of the students in both class periods for the three assessments administered during the unit, quiz 1, quiz 2, and the unit test. I will also discuss the students responses to the Google Form in both class periods both pre- and post-instruction. All of this data will be compared between class periods to show the differences between students’ performance with and without metacognitive strategies and self-regulation.

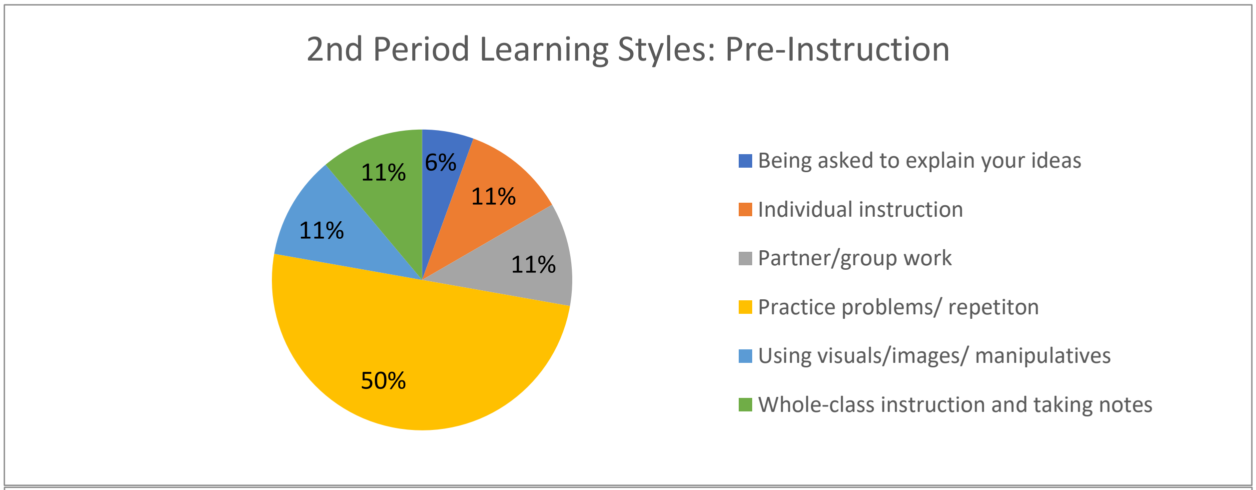
Before beginning this research, I looked at the students’ average scores on the previous unit tests for chapters one and two to determine if the classes of students are similar or different in their performance, thus far in their coursework. I wanted to ensure that when comparing the scores of these two classes post-instruction in my unit there wasn’t a class that was already performing higher than the other, which would skew my results. When comparing the scores for both unit tests for both classes the classes performed on average within <1% of each other. On the next page are the tables that have all of the students' scores for both class periods on both tests as well as the average score for each class period.

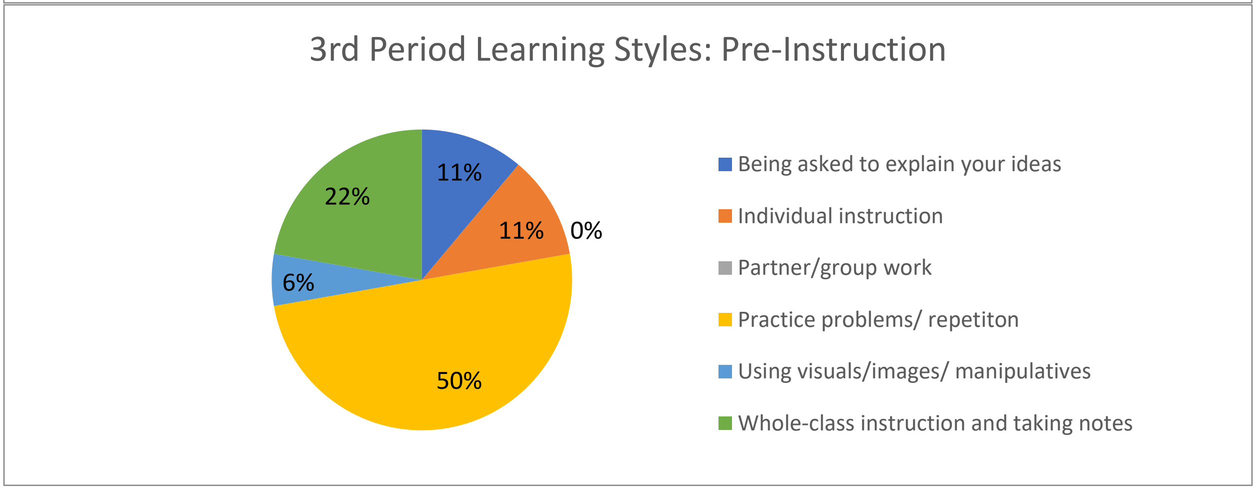
Chapter 1 Unit Test	
3rd Period	2nd Period
92	90
60	82
82	64
63	54
58	69
79	75
70	55
63	67
50	69
82	75
81	78
63	56
64	65
61	71
70	68
58	80
78	64
59	63
Average:	Average:
68.5	69.16666667

Chapter 2 Unit Test	
3rd Period	2nd Period
95	85
61	92
84	73
56	64
72	65
94	72
57	57
71	69
84	67
91	79
73	79
55	56
56	65
71	73
60	67
81	75
49	65
70	69
Average:	Average:
71.11111111	70.66666667

Based on this comparison the students in both class periods perform at approximately the same level on the previous two unit tests, again within <1% of each other. This makes comparing their scores after the unit 3 instruction that much more meaningful if there is a significant difference in the average scores. Furthermore, after these two tests based upon the average scores students in both classes are competent in their mathematics skills. Looking at all of the scores on each of the unit tests they all resemble a bell curve shape. The standard deviation for 3rd period and unit test 1 was 11.29%, using this standards deviation 13 of the 18 students in 3rd period, or 72%, scored within 1 standard deviation from the mean. Furthermore, 17 of the 18 students in 3rd period, or 94%, scored within 2 standard deviations from the mean. The standard deviation for 2nd period and unit test 1 was 9.65%, using this standards deviation 12 of the 18 students in 2nd period, or 67%, scored within 1 standard deviation from the mean. Furthermore, 17 of the 18 students in 3rd period, or 94%, scored within 2 standard deviations from the mean. The standard deviation for 3rd period, and unit test 2 was 14.4%, using this standard deviation 13 of the 18 students, or 72%, scored within 1 standards deviation of the mean. Furthermore, 18 of the 18 students in 3rd period, or 100%, scored within 2 standard deviations from the mean. The standard deviation for 2nd period and unit 2 test was 9.09%, using this standard deviation 14 of the 18 students, or 78%, scored within 1 standard deviation of the mean. Furthermore, 17 of the 18 students in 2nd period, or 94%, scored within 2 standard deviations from the mean.

At the beginning of the unit students in both class periods completed the Google Form answering the question of "How do you personally prefer to learn new mathematical content?". Students chose one of the following: Using visuals/images/manipulatives, being asked to explain your ideas, practice problems/repetition, individual instruction, partner/group work, and whole-class instruction and taking notes. Below are pie charts that represent the percentage of students who choose each category in both the second and third period classes pre-instruction.





Looking at the pie charts above both classes have a 50% majority of students who prefer to learn new mathematical content using practice problems and repetition. I used this information to design our extra practice day in third period's instructional plan, as well as to assign students homework problems to engage in practice and repetition. In the third period class there is also 22% of students who prefer taking notes in a whole-class instruction format, which I also made sure to incorporate into my unit outline on some days, while still engaging students in metacognitive questioning strategies. Furthermore, both classes had 11% of the students who preferred individual instruction, being asked to explain their ideas. For second period 11% of students preferred using visuals/images/manipulatives, while only 6% preferred so in 3rd period. Additionally, none of the students in the third period class prefer partner/group work and 11% of the students in 2nd period preferred this method. Despite this information, I did incorporate partner/group work into my unit plan, with the purpose of encouraging students to engage in meaningful mathematical discussion. This type of mathematical discussion was intended to develop students ability to explain their thought processes and defend them to their peers, all to deepen their understanding of the content. This partner/group work was only utilized in 3rd period as another implementation method of metacognitive questioning and discussion all in an effort to see how these changes affected the students' performance on the unit test and their opinions on their preferred mathematical learning style.

The students in both classes took the same two quizzes, attached as Appendices C and D respectively, during the instruction period of this unit and their scores on these quizzes revealed interesting things about their progress. The first quiz covered the definition of a function, function notation, domain and range visually, and defining the domain of a function from its equation. While the second quiz covered function notation, domain and range visually, defining the domain of a function from its equation, as well as piecewise functions. The third period students scored significantly better on average, on both quizzes. Below are the tables with all of the students' scores for both class periods and both quizzes.

Chapter 3 Quiz 1	
3rd Period	2nd Period
95	76
86	95
90	40
62	40
70	76
100	95
90	86
86	81
81	71
100	62
72	90
60	90
60	33
62	95
60	50
81	81
90	71
78	70
Average:	Average:
79.05555556	72.33333333

Chapter 3 Quiz 2	
3rd Period	2nd Period
100	98
85	70
98	58
48	73
88	70
95	83
55	78
50	70
93	83
85	93
90	73
38	63
98	80
55	90
90	61
83	90
78	35
79	73
Average:	Average:
78.22222222	74.5

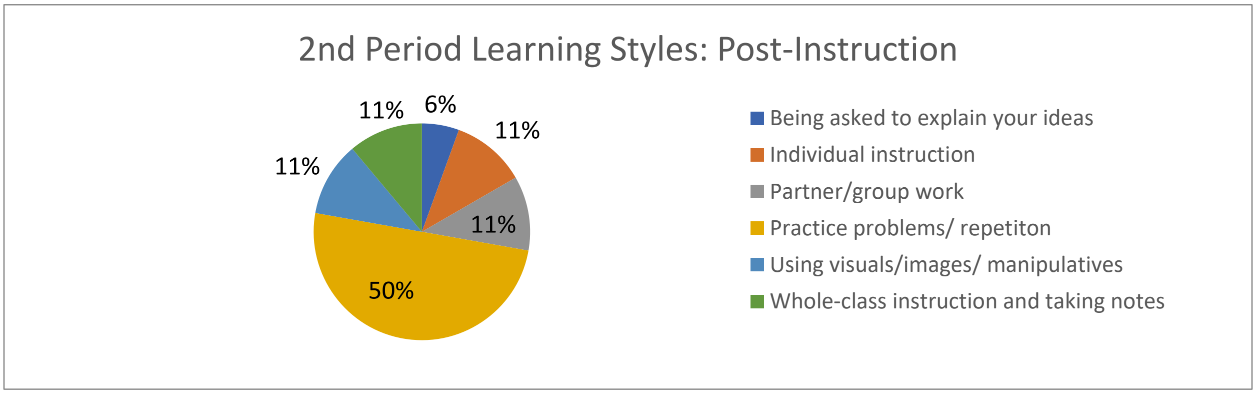
The students in third period performed > 6% better on average on the first quiz and > 3% better on average on the second quiz. The students in second period did not engage in the metacognitive questioning, goal setting, discussion-based activities, and metacognitive discussions that the students in third period experienced as I described in the methodology section. The only difference in the instruction, homework, and assessments administered to both of these class periods was the differences in instruction described in the methodology section. The students in second period experienced mostly direct instruction rather than the complex activities third period experienced.

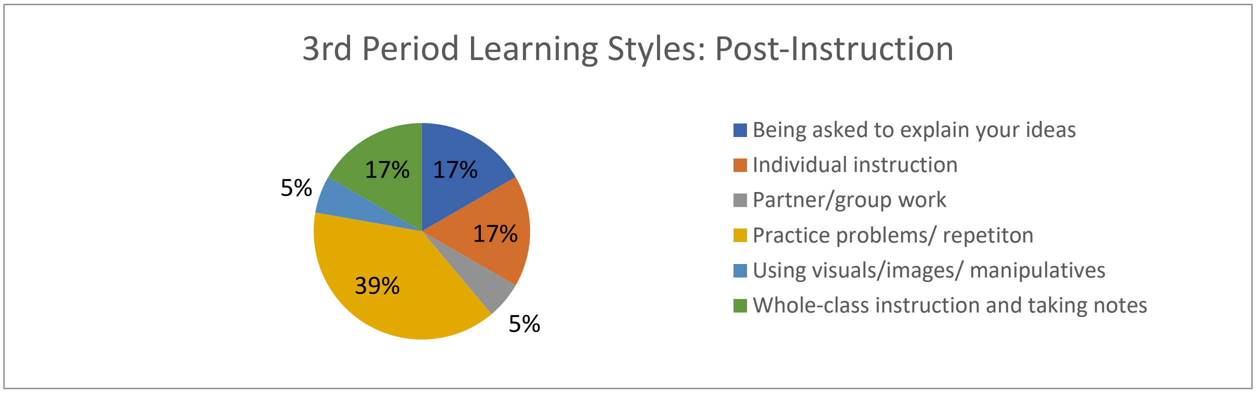
Looking at both class period's scores on the unit test the students in 3rd period scored on average > 4% better on the unit 3 test than their peers in 2nd period. The students in 3rd period experiences the same homework assignments, mid-unit assessments (quizzes), and summative assessment (unit test) as their peers in 2nd period. The students in 3rd period experienced lessons involving metacognitive questioning, small group and whole-class discussions, defending their answers and thought processes to their peers, goal setting, and the incorporation of self-regulation strategies. Therefore, incorporating these things into the instruction made a difference in the students' performance both on the quizzes discussed previously but also on their unit test, for which the scores are shown below:

Chapter 3 Unit Test	
3rd Period	2nd Period
92	88
79	86
91	73
55	36
71	79
97	73
61	77
83	68
88	73
79	80
44	61
44	60
67	82
85	65
73	62
91	64
70	86
74	55
Average:	Average:
74.66666667	70.44444444
Standard Deviation:	Standard Deviation:
15.8596788	13.11587373

The students' scores for each class period, the average for each class period, and the standard deviation for each data set are in the table to the left. The scores of the students in 3rd period and 2nd period are both similar to a bell curve, 3rd period models a bell curve more accurately than 2nd period. Using the standard deviation for 3rd period, 11 of the 18 students, or 61%, scored within 1 standard deviation from the mean. Moreover, 18 of the 18 students in 3rd period, or 100%, scored within 2 standard deviations from the mean. Using the standard deviation for 2nd period, 15 of the 18 students, or 83%, scored within 1 standard deviation from the mean. Also, 17 of the 18 students in 2nd period, or 94%, scored within 2 standard deviations from the mean.

Finally, looking at the students responses to the Google Form, Appendix A, post-instruction there were some changes in the students' responses, specifically the students from the students in 3rd period. Below there are two pie charts that represent the different percentages of students who chose each of the different learning styles after the conclusion of instruction.





The students in 2nd period all responded the same way to the Google Form, meaning there was no change in their preferred learning style. In 3rd period the number of students who prefer practice problems and repetition decreased and the number of students who prefer being asked to explain their ideas and partner/group work increased. The changes in these students preferred learning styles can be attributed to being exposed to explain their ideas in a discussion format during the unit as well as partner/group work to learn math. The students who changed their preferred choice to these categories, possibly enjoyed these experiences in class which caused this change in their preferred learning style. Furthermore, the number of students who prefer whole-class instruction and taking notes as well as individual instruction also increased. These students may have had positive experiences with the individualized goal setting and reflection in class or with the whole class individual activity days, which caused their change in their preferred learning style.

VI. Conclusions/Implications/Limitations

My research taught me a lot about my students, who I am as a teacher, and the implications of metacognitive strategies in a mathematics classroom. First, the largest message from this study is that students benefit from engaging in metacognitive discussion, metacognitive-styled learning, and self-regulation strategies. In other words, implementing these strategies worked.

The students in third period performed significantly higher on all three assessments during the unit. These students showed a better understanding of the content than their peers in second period through their performance on assessments and discussions during class. Furthermore, the third period students demonstrated an improved attitude toward learning math. Specifically, they were excited to come to class and experience these new learning/teaching styles each day. Finally, ~25% of the students in third period shifted their preferred mathematics learning style after the completion of the unit. This means that the experiences these students had during the instruction helped them to better understand themselves as learners, and they adjusted accordingly. Furthermore, I learned that as a teacher I truly enjoy engaging students in discovery-based learning as well as discussions about mathematics and their reasoning with their peers. I see both of these teaching strategies as vital parts of who I am as a teacher, and I have discovered this through my research.

I also learned that students would rise to meet the high expectations you have for them, but you must provide them with the tools they need individually. Multiple students in both classes had difficulties engaging in some of the discussion and/or discovery-based activities because this was an unfamiliar experience for them in a mathematics classroom. I learned that I need to be overly prepared to assist these students with pre-prepared questions, extra tools, and pre-determined grouping to ensure that all of the students benefit from these strategies.

My findings through this study were overall fairly consistent with past research that was described in my literature review. The students in the class where the metacognitive strategies and self-regulation strategies were implemented, performed significantly better on average on their final unit test and the two quizzes during the instruction of the unit. Moreover, there were more students in third period who changed their preferred mathematical style, based on the results of the Google Form. These students experienced new teaching/learning styles and thus were able to re-evaluate their preferred mathematical learning style based on their experiences.

While my findings were consistent with the research from my literature review, there were still some limitations to this study. First, the instruction was delivered during the COVID-19 pandemic where students were to remain socially distanced in classrooms and were required to wear masks at all times. These circumstances made group work and collaboration more difficult because while students were grouped together, they weren't allowed to move their desks at all and only limited social cues could be seen. Furthermore, due to the COVID-19 pandemic multiple students in both class periods were absent during one or multiple days of instruction and attended virtually or not at all. This impacted their understanding of the content and ability to engage in some of the activities. Finally, this instruction was only delivered over a two-week period, and it was many students' first experience with metacognitive questioning strategies, mathematical discussions, and self-regulation. Thus, the impact of these strategies may have been diminished.

Ultimately, if I could continue this study with other students in another classroom, I would begin by having students complete a survey about their experience and knowledge about metacognition in a mathematics classroom. Next, I would dedicate a portion of the first day of instruction to teaching students about these strategies and setting clear norms/expectations. I would implement more metacognitive questioning strategies in exit-ticket form and more accurately record the students responses to be analyzed later. Finally, I would complete the study over a longer period of time, through multiple units of instruction, and compare students' scores on each unit test to assess their progress in understanding and using metacognition in the mathematics classroom.

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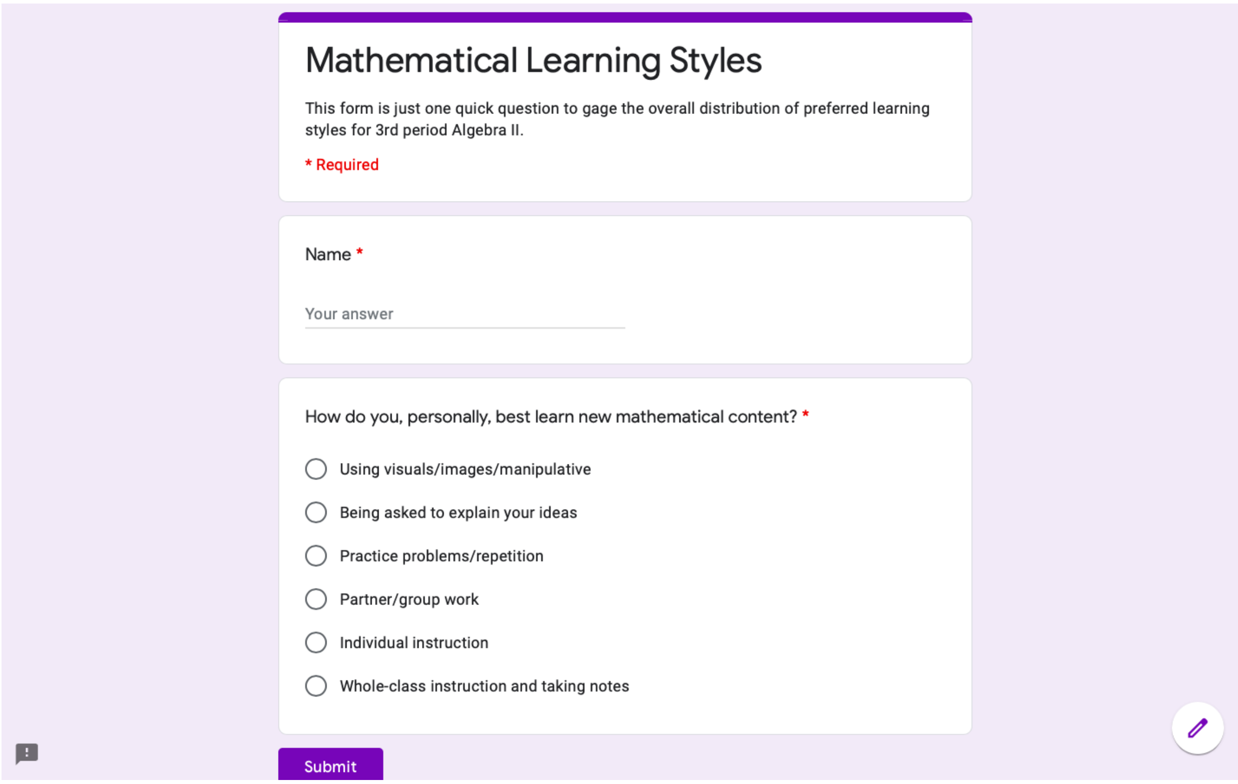
Appendix A

Google Form – Mathematics Learning Styles



Mathematical Learning Styles

This form is just one quick question to gage the overall distribution of preferred learning styles for 3rd period Algebra II.

* Required

Name *

Your answer

How do you, personally, best learn new mathematical content? *

- Using visuals/images/manipulative
- Being asked to explain your ideas
- Practice problems/repetition
- Partner/group work
- Individual instruction
- Whole-class instruction and taking notes

Submit

Appendix B

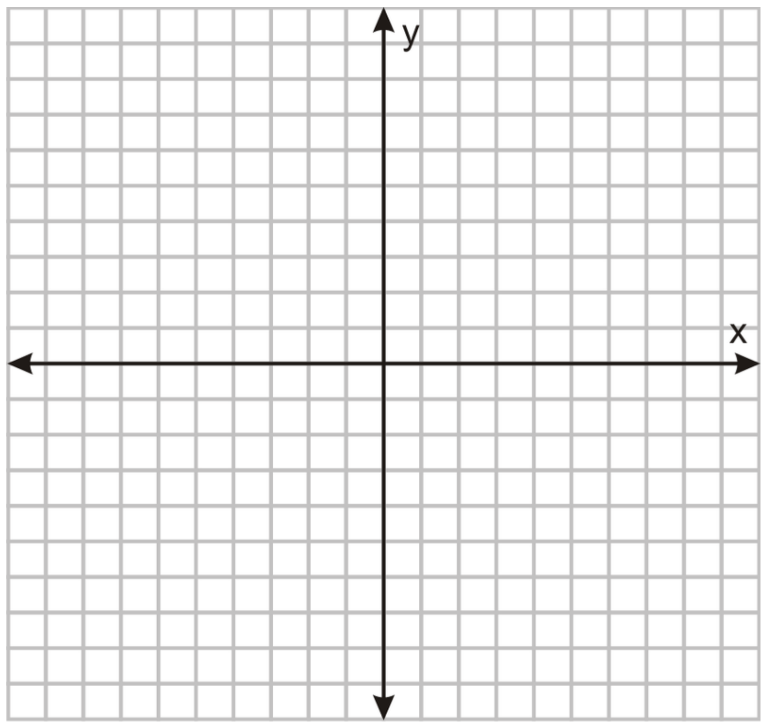
Algebra 2A – 3.1 Exit Ticket

Algebra 2A Name: ______________________

3.1 Exit Ticket

Using the equation $y = \frac{3}{4}x - 2$ answer the following questions:

1. Graph the function on the coordinate graph below:



2. Is this relationship a function? Explain why or why not in detail.

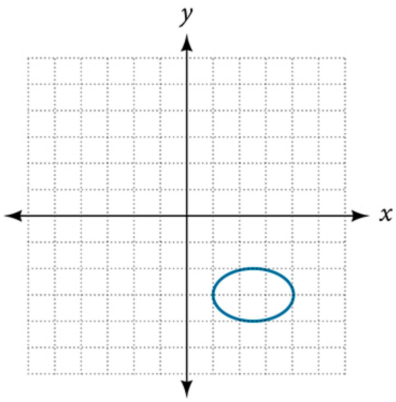
Appendix C

Algebra 2A – Unit 3 Quiz 1

Name: ___________________________

Determine if the following relationships are functions or not:

1.



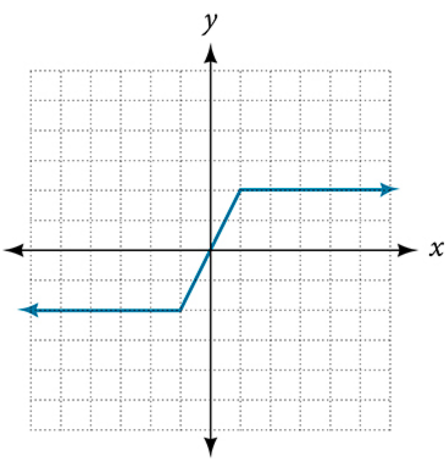
____

2.

Input	Output
-3	5
0	1
4	3
7	4
9	5

____

Using the graph below evaluate the following:



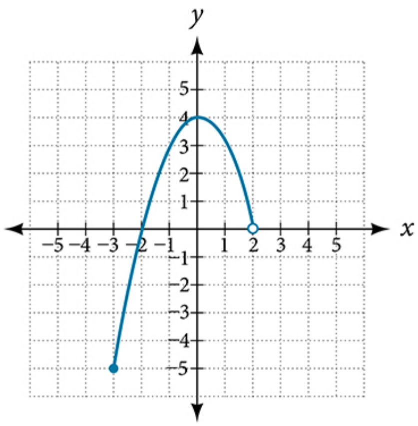
3. $f(2) =$ _________

4. $f(-4) =$ _________

5. $f(x) = 0 \quad x =$ _________

6. Domain: _________________ Range: __________________

Using the graph below evaluate the following:



7. $f(1) =$ ________

8. $f(x) = -4 \quad x =$ ________

9. Domain: ________________ Range: _________________

Define the domain of the following functions using just their equations:

7. $f(x) = \frac{1}{x-1}$ ________________________

8. $g(x) = \sqrt{2x - 4}$ ________________________

Appendix D

Algebra 2A – Unit 3 Quiz 2

Name: ________________________

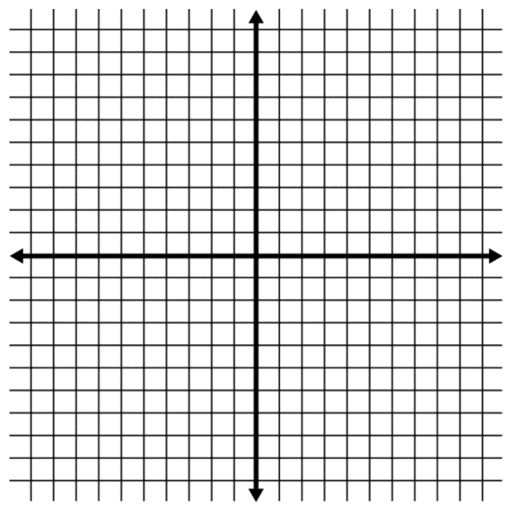
Evaluate the functions: $f(x) = x^2 - 3x,\ g(x) = \frac{x+4}{3x+6}, h(x) = \sqrt{3x-5}$ at the given values:

$f(3) =$ ______ $h(7) =$ ______ $h(-5) =$ ______

$g(2) =$ ______ $f(-4) =$ ______ $g(-8) =$ ______

Graph the following piecewise function and evaluate the function at the given values

2. $h(x) = \begin{cases} x+2\,,\ x < 2 \\ -1/2x + 3\,,\ \ x \geq 2 \end{cases}$



Evaluate:

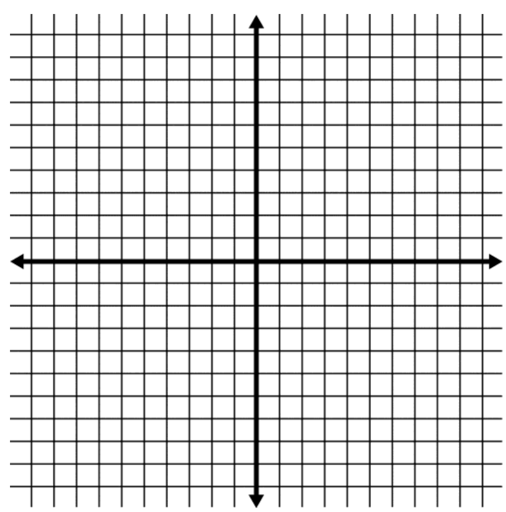
a. $h(4) =$ ________

b. $h(x) = -5\,; x =$ ________

c. Domain: _____________

Fill in the domain for the piecewise function and evaluate the function at the given values

3. $g(x) = \begin{cases} x^2 - 4\,, \text{\_\_\_\_\_\_\_\_} \\ \frac{1}{3}x + 3\,,\ \ \text{\_\_\_\_\_\_\_\_} \end{cases}$



Evaluate:

a. $g(-3) =$ ________

b. $g(x) = 6\,; x =$ ________

c. Range: _____________

Define the domain of the following functions using just their equations:

4. $f(x) = \frac{(x+1)}{x^2-3x-4}$ ______________________

5. $g(x) = \frac{x-2}{x^2+7x+12}$ ______________________

6. $h(x) = \frac{1}{\sqrt{x^2-10x+25}}$ ______________________

7. $f(x) = 2x + 3$ ______________________

8. $f(x) = \frac{x+2}{x-5}$ ______________________

9. $f(x) = \sqrt{3x-4}$ ______________________

Appendix E

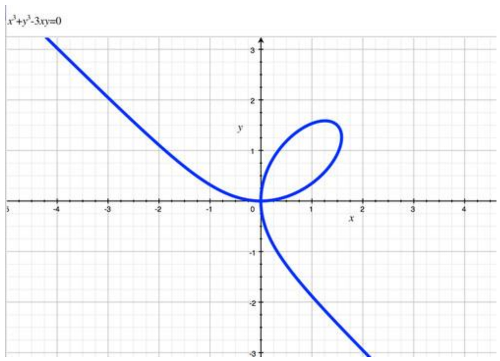
Unit 3 Test

Algebra 2A
3.1 – 3.5 Test

Name: ______________________

1. Do the following graph and table represent a function?

a.

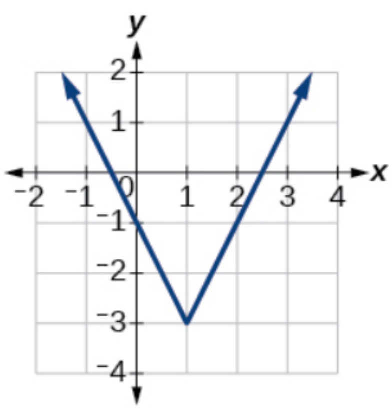


b.

Time (s)	Height (m)
0	7
2	10
4	5
6	0
7	0
8	3

____________ ____________

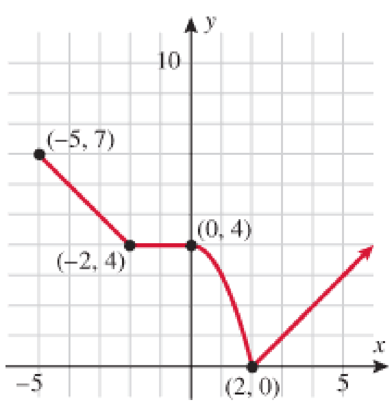
2. Define the domain and range of the function below and evaluate it at the given values



Domain: ______________________

Range: ______________________

$f(-1) =$______ $f(2) =$______



Domain: ______________________

Range: ______________________

$2f(0) + 3f(-4) =$________

3. Use the functions $f(x) = 2x + 7$, $g(x) = x^2 + 7x + 12$, and $h(x) = x^2 + 6$ to evaluate the following:

$2h(-3) =$__________ $2f(3) - \frac{1}{2}g(1) =$_________ $f(-9) \cdot h(4) =$__________

4. Define the domain of the following functions using their equations:

$f(x) = \frac{x-3}{x^2-12x+36}$ ________________________

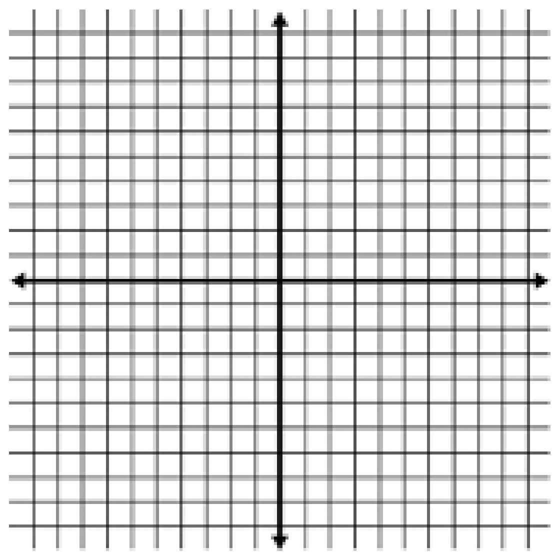
$f(x) = \frac{\sqrt{x-4}}{x-9}$ ________________________

5. Graph the following piecewise function & evaluate it at the given values:

$$f(x) = \begin{cases} 3x - 3, & x < 3 \\ -x + 1, & x \geq 3 \end{cases}$$

a. $f(3) =$_______

b. $f(-1) =$_______

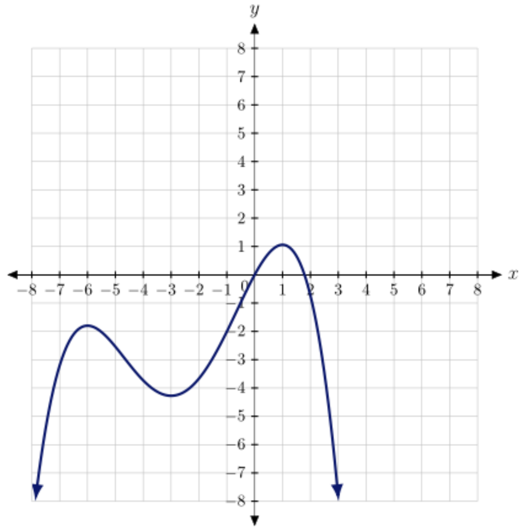


6. Find the average rate of change of the following functions on the given intervals:

a. $f(x) = \frac{3x-5}{2-x}$ on [-3, 3] ________________________

b. $g(x) = x^2 + 3x - 11$ on [-5,2] ________________________

7. Find the following using the graphs:

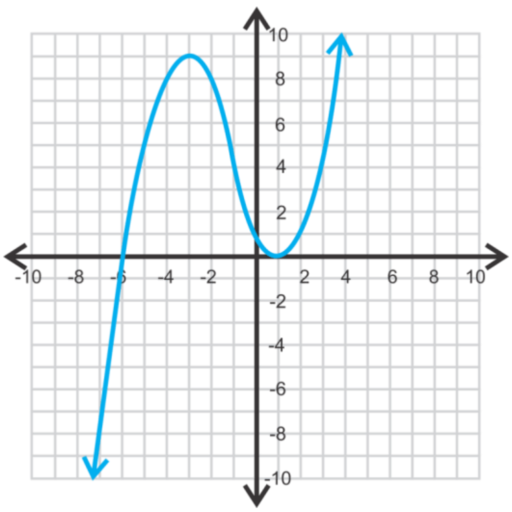


a. Absolute Maximum: ______________________

b. Local Maximums(s): ______________________

c. Domain: _______________

d. Range: _____________



Interval(s) of Increasing: ______________________

Interval(s) of Decreasing: ______________________

Interval(s) of Constant: ________________________

Domain: ____________ Range: ______________

8. Use the functions $f(x) = x^2 + 5x + 4$ and $g(x) = \frac{6x+4}{2x}$ to find the following:

$g(f(x)) =$ ______________________ $f(g(12)) =$________

$f(g(1)) =$ ______________ $f + g(-2)) =$________