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Eighth-grade Students' Use and Justification of Multiple Represenations

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Honors Project

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The purpose of this study is to explore students' use of multiple representations in the classroom and their reasoning for representation choice. The participants of this study were 22 eighth-grade students. All students took a short pretest consisting of three items. The items addressed mathematics content covered during regular instruction. Students were asked to answer each question using different representations. Usual instruction included frequent probing and encouragement of students' representation use while problem solving. Students were asked to explain their reasoning when problem solving and share their representations with others. Following the unit of instruction, students completed a posttest composed of the same items and directions from the pretest. Students showed an increase in number of representations used as well as number of test items answered correctly.

Introduction

Effective mathematics teaching challenges students to use varied forms of representation and questions student reasoning for representational form (National Council of Teachers of Mathematics [NCTM], 2000; 2014). Mathematics teachers should strive to encourage students to use a variety of representations while problem solving (NCTM, 2014; 2000). Moreover, they might also aim to understand why students choose to use various forms of representation. Teachers can adjust their instruction in ways to engage all students by having a better understanding of students' reasons for representing their problem-solving solutions. There are a variety of ways to represent solution strategies; this variety helps to clarify underlying mathematical concepts by showing different aspects of the concept (Triphathi, 2008). According to NCTM, "Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving" (2014, p. 24). If students engage in the use of multiple representations then they may begin to connect these representations of mathematical ideas, which, in turn, fosters sense making of mathematics correctness (Yee & Bostic, 2014). Students who use multiple representations are shown to be better problem solvers than those who only use one form of representation (Bostic and Pape, 2010; Herman, 2007; Perry and Atkins, 2002; Preston and Garner, 2003; Tripathi, 2008). This improvement in problem solving may be linked to using "different representations [which] is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture [concept] richer and deeper" (NCTM, 2014, p. 25). It is clear that the use of multiple representations helps students learn and teachers should try to incorporate the use of multiple representations into their classrooms. The purpose of this study is to explore the use of multiple representations in the classroom and try to understand students' reasoning for representation choice. It is hypothesized that the use of multiple representations will increase students' problem solving abilities.

Literature Review

The purpose of my action research is to investigate why students select representational forms while problem solving. It is important to recognize various types of background knowledge, including what constitutes a representation, how teachers can implement multiple representations into their classrooms, how multiple representations affect student learning, and why students choose to use multiple representations. For the purpose of this study, "representation" is defined as the way a mathematical concept is organized, recorded, and communicated (NCTM, 2000). Representations will be categorized as "symbolic" and "nonsymbolic" as in past research (see Goldin & Kaput, 1996; Yee & Bostic, 2014). Symbolic representations are abstract, symbol-driven ways of expressing oneself that include forms such as expressions, equations, and inequalities (Goldin & Kaput, 1996). Nonsymbolic representations take the form of anything that is not symbolic such as diagrams or pictures, graphs, tables, spoken language, and concrete models (Lesh & Doerr, 2003). Constructing multiple representations is characterized as the act of using two or more forms of representation to arrive at a solution.

Representation and Re-Presentation

When working with multiple representations, it is important to understand the act of re-presentation. Re-presentation is the act of abstracting a representation and being able to spontaneously re-present the information in the form of a different representation (Von Glasersfeld, 1991). Representation is the initial method a student chooses to use to answer a problem. On the other hand, re-presentation involves working backwards from the original representation and being able to abstract the method in hopes of applying this strategy to other problems. While a student may be able to construct one representation, that fact does not mean that the student will be able to re-present the information in the form of another representation. According to Von Glaserfeld (1991), "The ability to recognize a thing in one's perceptual field, however, does not necessarily bring with it the ability to re-present it spontaneously." Students learn how to read and interpret representation but it is the act of re-presenting the material that strengthens students' understanding and builds representational fluency (Heinze, Star, Verschaffel, 2009). In order to re-present the information, students must first abstract the representation so that they can reconstruct the information in a different representation. When students are able to re-present the information in the form of another representation, they have a better understanding of the information.

Multiple Representations: Characterizations and Examples

Multiple representations can be constructed in two ways (Herman, 2007). The first form of multiple representations is when a person constructs one representation and then another. Both representations are constructed independently but represent the same concept. One example of this form is if a student is asked to solve the equation $2 \div \frac{1}{2} =?$ One student may choose to solve the problem algebraically by noticing how dividing by one-half is the same as multiplying by 2. Thus, $2 \div \frac{1}{2} = 2 \times 2 = 4$. The student may try to represent this idea in another way because he/she may wonder why dividing by one-half is the same as multiplying by two. The student may choose to solve this problem using fraction tiles. In this case, the student would set the problem up by finding two whole circles, see Figure 1; the student would then overlay $\frac{1}{2}$ tiles (i.e. semicircles) to show dividing by $\frac{1}{2}$, see Figure 2. It may be clear to the student that $2 \div \frac{1}{2} = 4$ because he/she can lay four one-half fraction tiles on top of two whole fraction tiles. Both of these representations were created separately but they represent the same idea.

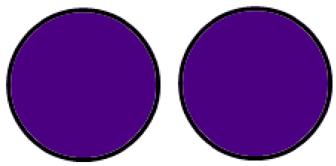

Figure 1. Whole Fraction Circles

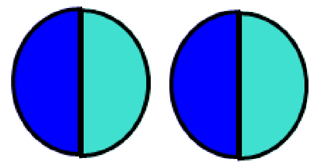

Figure 2. Half Fraction Circles

The second form of multiple representations is when two or more representations are constructed concurrently, as seen in Bostic and Pape (2010). These representations work hand-in-hand and therefore it makes sense that they should be co-constructed. An example of this form of multiple representation use might be creating a symbolic equation and graphing the equation at the same time. For example, if a student wanted to solve for "x" in the equation $x^2 + 5x + 4 = 0$, then the student may start by solving the problem symbolically: $x^2 + 5x + 4 = 0$ => $x^2 + 1x + 4x + 4 = 0$ => $x(x + 1) + 4(x + 1) = 0$ => $(x + 4)(x + 1) = 0$ => $x + 4 = 0, x + 1 = 0$. Thus, $x = -4, -1$. The student may simultaneously graph the equation and look at the x-intercepts. See Figure 3.

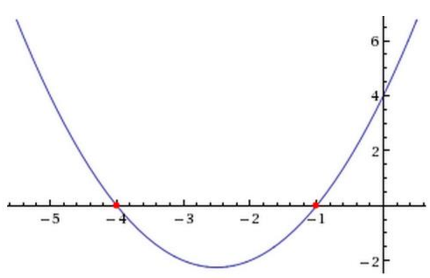

Figure 3. Graph of $y = x^2 + 5x + 4$ (Wolfram Alpha, 2015)

The graph clearly shows that y=0 when x=-4, -1. This confirms the student's original conclusion that x=-4, -1. The use of both representation forms helps the student confirm and understand the mathematics, which are found in this problem.

Teaching and Learning with Multiple Representations

One common theme throughout the literature on multiple representations is the idea that students obtain a deeper understanding of the content when they use multiple representations. Herman (2007) believes that addressing algebraic problems through the use of multiple representations supports students' understanding of the concepts involved in the problems, particularly when students build connections across the different representations. Students form connections when they see how one representation is incorporated or represented in another representation. For example, when a student algebraically solves a binomial to find the roots and then graphs the function to see the roots graphically, the student is seeing where the roots show up in the graph. The connection between the algebraic and graphical representations helps the student to understand that a root occurs where the graph crosses the x-axis. When students begin to see how one representation connects with another, they start to construct a big picture view of the underlying mathematical concept. This abstracted view of the mathematical concept allows students to solve problems without relying on procedures but instead on their own understanding of the concept. This idea was further explored when Piez and Voxman (1997) noted that because each representation emphasizes and suppresses various aspects of a concept, students gain a more thorough understanding of a function if it is explored using numerical, graphical, and analytical methods. Piez and Voxman (1997) shared the value of multiple representations in precalculus and calculus classrooms; Bostic and Pape (2010) found similar outcomes in four algebra II classrooms. Bostic and Pape (2010) offer results indicating that when students engaged in instruction emphasizing multiple representations, they were more likely to succeed in finding the problem's solution and performed better than their peers on mathematical tests of functions presented in a word problem format. Not only do students perform better on problem-solving tasks when they work with multiple representations, but they also enjoy working with more than one representation. Herman (2007) found college algebra students recognized benefits of learning many ways to approach a problem and thought that using multiple representations deepened their understanding. According to those college algebra students, another good reason for knowing more than one way to approach a problem is to have a repertoire of strategies from which to pick. When students integrate across representations, they begin to understand the mathematical concept from various perspectives, which helps them to understand the concept better as a whole. If students understand the benefit of multiple representations and if using multiple representations is beneficial to students' problem-solving abilities, then it logically follows that teachers should strive to incorporate multiple representations into the classroom.

Many researchers have found ways to incorporate multiple representations into the classroom and have recorded their findings. An emergent theme across the literature is finding an activity or problem that allows students to represent solution strategies in multiple ways. In a study conducted by Preston and Garner (2003), seventh-grade students were given the task of determining which of three different companies offered the cheapest price to plan a party. Students were not given how many people would be attending the party but were given the cost of each facility based on the number of participants. They worked in groups to determine the best facility at which to have the party. Students expressed that using multiple representations helped them during problem-solving because it gave them varying perspectives of the problem.

These varying perspectives allowed students to effectively solve the problem. Similarly, Tripathi (2008) gave her middle schoolers a problem that involved a farmer who had 19 animals, chickens and cows, and a total of 62 legs on the animals on the farm. The question prompted students to determine how many of each animal the farmer had. Tripathi's problem was an excellent one for using multiple representations because students could use graphs, pictures, and tables to solve the problem. Tripathi (2008) suggested giving students problems that encourage them to employ visual representations. Visual representations support and illustrate symbolic results, resolve conflicts between correct and incorrect solutions, and show conceptual underpinnings that symbolic representations may not make explicit (Tripathi, 2008). It is clear that multiple representations can be used in classes ranging from first grade all the way to high school calculus based on previous research that was conducted. Teachers should take time to find tasks that allow students to use multiple representations while problem solving.

Another important aspect to consider when teaching with multiple representations is asking students to explain their representations. Perry and Atkins (2002) asked first- and fourth-grade students to represent block towers in any way they wished; representations ranged from drawings to verbal explanations. Perry and Atkins (2002) found that just because the teacher thought the representation told her what the student knew, it was not until the teacher discussed the representation with the student that the teacher truly understood the student's understanding of the mathematics concept. When students explain their representations, then their explanation helps solidify students' understanding of the mathematical concept (Perry & Atkins, 2002). This verbal re-presentation is what Von Glasersfeld (1991) suggests strengthens students' understanding. Additionally, students have to abstract their initial representation in order to re-present it as a verbal explanation. Even though Perry and Atkins worked with first- and fourth-grade students, Preston and Garner (2003) also suggested having seventh-grade students explain their representations because each student's explanation allows other students to understand the representation better. Some students used different representations, from the ones they used to solve the problem, to explain their ideas to the class. If teachers are going to incorporate the use of multiple representations into their classrooms, then they must consider two questions: what problems will allow for students to use multiple representations and what questions do they need to ask to understand the students' representation. Based on prior research, it is clear that using multiple representations in the classroom is both feasible as well as beneficial to students' learning.

Rationales for Representations

Teachers using multiple representations should ask students to explain their representations in order to understand the students' thought process (Perry & Atkins, 2002). When students share their representations, other students are exposed to new ways of approaching problems, which they can implement in the future (Herman, 2007). Students become better problem solvers by viewing and understanding multiple strategies, which can include but are not synonymous with using multiple representations (Bostic, Pape, and Jacobbe, 2016). Teachers can engage their class in the use of multiple representations by taking the time to ask for various forms of representation. There is quite a bit of research on students' use of multiple representations in the classroom as well as the benefits from using multiple representations during instruction. What is not clear from the literature is why students choose to use certain forms of representation. Piez and Voxman (1997) asked high school students to explain why they chose to work with a particular method when solving a problem. However, Piez and Voxman were really asking students about a rationale for a particular procedure and not necessarily their representation or why they chose a specific form of representation. It is important to note that a procedure is different from a representation (Lesh & Doerr, 2003). Procedure refers to a student using some method or formula, given to him/her to solve a problem with no context for why the formula works. Representation is used when students solve a problem in whichever way they see fit; they are not instructed on how to solve the problem. Another distinction between procedure and representation is that a procedure has a focus on the goal of finding the correct answer whereas representation focuses on the process of finding an answer. In my research, I explored students' rationales for their representations, with a keen eye on students' use of multiple representation while problem solving.

Research Question

The focus of my research was students' rationale for their representation use. My research question was: Why do students choose certain forms of representation when engaging in problem solving?

Method

I employed a mixed-methods approach (Cresswell, 2012) to answer my question: Why do students choose certain forms of representation when engaging in problem solving? Both quantitative and qualitative data were gathered. The qualitative data were used to help explain the results of the quantitative data.

Instruction included frequent probing and encouragement of students' representation use while problem solving. Students were asked to explain their reasoning when problem solving and share their representations with others.

Intervention

After the pretest was administered and three students had been interviewed, a two-week period of instruction began during which the teacher gave students opportunities to explore multiple representations. On the first day of instruction, students engaged in an activity in which they defined what it meant to represent and listed examples of various representations. This activity gave students a base line understanding of what representation was and what types of representation could be utilized in class. Also, on the first day, students physically represented an exponential growth function to help students begin thinking about exponential growth functions. The first week of instruction had a focus on exploring various representations to allow students to become familiar with the various representations. One day students were asked to provide examples of three types of representations. During instruction, the focus was put on having students represent and solve exponential functions in as many ways as possible. Students were asked to share their ideas with the class to increase the number of representations students were exposed to (Preston and Garner, 2003). Students were asked to compare linear and exponential functions each represented differently and determine which had the larger value. This activity allowed students to see the benefits and drawbacks of the different types of representations. The second week of instruction still asked students to explore exponential growth functions using multiple representations but additional conversations happened about representation in a general sense. Students were asked to explain how one representation (i.e., table, graph, or equation) could help explain another representation. Additionally, students were asked to discuss the benefits of using multiple representations.

Instruction concluded with a group project in which students were asked to represent and solve an exponential growth function. Each group was assigned a different reward plan that represented the amount of money a peasant would make for a reward. The students were asked to employ as many representations as possible to convince the class that their plan would help the peasant acquire the most money. Each group was required to present their plan to the class and justify why each representation helped explain why their plan was the best. Furthermore, each student was required to determine which plan was the best for the peasant using evidence from the representations. This activity asked students to compare across representations and use multiple representations to create a big picture understanding of the given plan. After the two weeks of instruction students completed a posttest consisting of the same test items as the pretest and interviews were conducted.

Data Collection

Quantitative

The participants of this study were 22 eighth-grade mathematics students. These students came from one section of eighth-grade Mathematics II. Students in Mathematics II are middle ability students. The study took place during the second half of the academic year. All students took a pretest consisting of three items (see Appendix A). The items addressed mathematics content taught during instruction. Each item asked the students to answer the question using as many representations as they could. Students answered each question in multiple ways using different representations, much like others (see Bostic & Pape, 2010; Herman, 2007). Following the unit of instruction, students completed a posttest composed of the same items and directions from the pretest.

Qualitative

Upon completion of the pretest, three students were interviewed. The students were selected based on the number of representations they averaged on the pretest. The interview allowed students to elaborate on their answers from the pre- and posttest and answer questions such as: why did you use this representation first? Do you normally use more than one representation? Would you have solved the problem using multiple representations if you were not asked to? Would you change how you approach a similar problem in the future? The purpose of the questions was to explain why the students chose certain types of representations and why they engaged in multiple representations. After the posttest, the same three students were interviewed and were asked to discuss forms of representation they used in their pre- and posttests. I also inquired about why students chose a particular representation as their initial approach and whether they learned from using a second or third approach.

Data Analysis

To interpret the data, the study used an explanatory mixed-method approach where qualitative analysis was used to support the findings from the quantitative analysis in an attempt to explain the quantitative results.

Quantitative

All 22 students completed the pretest which was composed of three items where the students were asked to answer questions using multiple representations. The average number of representations each student used per item was calculated for comparison. Also, the representations were categorized by types (i.e. table, graph, and equation) in order to determine the most frequently used type of representation for each item and the pretest as a whole. At the end of a series of lessons, a posttest was given. The posttest consisted of the same three questions as the pretest. After the posttest, similar data was collected including the average number of representations used by each student per item. Additionally, a comparison was made between the average number of representations on the pretest and posttest as well as the number of correct responses.

Qualitative

Again, an interview happened including the original three students who were interviewed to see if their thoughts had changed on multiple representations and their reasoning for using certain representations. The interviews helped support the quantitative results.

Data were analyzed qualitatively using inductive analysis (Hatch, 2002). The interviews were recorded and listened to for general themes. Initially, the data were listened to closely. Secondly, notes were made of participants' responses to help outline student ideas. General themes were identified as concepts and points students made frequently throughout their interviews. Any ideas that were only mentioned with no explanation were dismissed. Key ideas all three students discussed were generalized and recorded as general themes. The themes will help to explain the quantitative results.

Results

Quantitative

Overall, there was an increase in both the number of representations students' used and the number of correct answers students had from the pre- to the posttest. This shows growth in both students' understanding of representations for solving problems related to exponential growth functions. It was conjectured that students might have been able to use more representations for item 1 than item 3 but still, every item could have been solved with at least two representations. A discussion of the pretest results is shared followed by results of the posttest.

The results of the pretest are reported below in Table 1. The number of representations used per item as well as the number of correct responses are shown.

Table 1

Pretest Results

Test Item	Total number of representations used	Mean	Standard Deviation	Total number of correct responses	Mean	Standard Deviation
Item 1	32	1.45	.67	7	.31	.48
Item 2	23	1.04	.49	0	0	0
Item 3	20	.91	.53	3	.14	.35

N= 22

The pretest asked students to represent and solve exponential growth functions. The results from the pretest show that students knew very few strategies for representing a given exponential function. Many students only used a table to represent the exponential growth function. Also, if students could not represent the function using a table, then they tried to use a picture. Most students used non-symbolic forms of representation on the pretest and chose to use the same type of representation for the three items. The mean number of representations used per item across the pretest was 1.09. This means that on average students could only employ one strategy for representing the exponential growth function. Since the mean number of representations for item 3 is less than one, this indicates that some students did not even attempt to answer question 3.

Similar results were seen regarding the number of correct answers students had on the pretest. Many students struggled to answer the questions; the mean score on the pretest was 15%, indicating that many students could not answer any item correctly. Item one had the most correct responses as compared to item two which had no correct responses. This was expected because students had yet to formally explore exponential functions, therefore, students did not know what an exponential function was.

Students took a posttest composed of the same three questions as the pretest after engaging in a two-week unit on exponential growth functions. The results are shown in Table 2.

Table 2

Posttest Results

Test Item	Total number of representations used	Mean	Standard Deviation	Total number of correct responses	Mean	Standard Deviation
Item 1	53	2.41	.67	13	.59	.50
Item 2	50	2.27	.77	4	.18	.39
Item 3	47	2.13	.77	10	.45	.51

N=22

The mean number of representations per test item increased from 1.09 to 2.27. On average, students used one additional strategy for solving the items covering exponential growth functions. The mean number of representations increased for every item by at least one unit. Students were able to find at least two representations for representing the problem as compared to only one strategy on the pretest. The total number of representations that students employed increased for all items.

The overall scores on the test improved from a mean of 15% on the pretest to 41% on the posttest. This means that on the pretest, most students could not answer one question but on the posttest students could at least answer one. There was growth across all items: Six more students answered item one on the posttest compared to the pretest, four more students answered item two on the posttest as compared to the pretest, and seven more students answered item three on the posttest as compared to the pretest. No students answered the second item on the pretest and four students answered it correctly on the posttest.

Qualitative

Three students were interviewed before and after the unit on exponential growth functions. The students were selected based on the number of correct responses on the pretest and the number of representations used per test item. Names are pseudonyms for participants in the action research study. One student, Alex, correctly answered two items on the pretest only using one representation per item. The second student, Reed, correctly answered one item on the pretest, using two representations per item. Finally, the third student, Olivia, did not correctly answer any pretest items, using two representations per item.

Pretest. It is reasonable to hypothesize that students employed fewer representations on the pretest because they did not know how to correctly represent exponential functions. A common theme from the pretest interviews was that students used a table as their first strategy for solving the problem. When asked why they used a table, none of the participants could elaborate on their reasoning. Moreover, none represented the exponential growth function using an equation. Alex said, "Normally, I use an equation but I didn't know how to make an equation for this problem." When asked if students might be able to employ more than one strategy to solve the item if they were not asked to do so, all three students said they would only use one strategy. Reed stated, "If you didn't ask me I would have just made a table because that answered the question." The students' statements aligned with students' pretest results. Put simply, they used fewer representations because they were not able to think of alternative representations.

Posttest. The same three students were interviewed to explore their use of representations as well as their justification for using them. When interviewing the three students after the posttest, a common theme emerged. Students employed more representations because they could explain the strategic benefits of multiple representations. All three participants started problem-solving by creating a table. When asked why they started with a table, Alex said, "I like to start with a table because it helps me organize my information so I can see the pattern." This was the same student that could not explain why he used a table first on the pretest. The other two participants made similar justifications for starting to explore the exponential growth problem with a table. Olivia stated, "I always start with a table because I like to see the pattern in the table first." After creating a table, two students decided to translate to an equation because it was easy to move from a table to an equation. The students expressed that the table allowed them to identify the growth factor and y-intercept they needed to create the equation. Students also shared that it was easy to answer the question once they had created the equation because they could evaluate the expression for the given value. Here, again, students were able to justify the benefits of using each solution strategy, hence their use of more representations on the posttest. Olivia was the only person who changed her mind about using multiple representations when not asked to. She stated, "I would probably use two representations to double check my work." All students agreed about the importance of using more than one strategy. "When you know more than one way to answer a problem you can always try a different approach if you are stuck," Olivia stated. Alex said, "If you use more than one strategy for solving a problem then you are really checking your work." Reed expressed, "When you use more than one way then it can help you to explain the problem better." Reed went on to explain that if he were trying to explain a problem to another student it would be helpful to know more than one strategy to solve the problem. He said not everyone learns the same way and that some people learn better from different representations. All participants could not explain the importance of multiple representations after the pretest, but after the posttest, their answers were clear and they each had different reasons as to why multiple representations are valuable. Students understood that using multiple representations provided them the opportunity to check over their work. Olivia recognized that knowing various approaches is beneficial not only for exponential functions, but also for problem-solving in general. Originally, Reed had stated that he would not use multiple representations, but, in the end, he could see the benefits.

Overall, the results showed that a focus on multiple representations during instruction improved students' ability to solve exponential growth problems and increased students' understanding and use of multiple representations. Also, as students began to understand exponential growth functions, they were able to come up with more representations.

Discussion

The purpose of this action research was to determine if emphasizing the use of multiple representations in the classroom would improve students' understanding of multiple representations and their reasoning for representation choice. The results of the study showed that after two weeks of instruction focusing on multiple representations, students used more representations for exponential functions, similar to Bostic and Pape (2010). Also, students could more clearly identify why they chose certain representations. Participants acknowledged the importance of using more than one solution strategy and could identify benefits of using more than one strategy. All material used for teaching this unit of instruction was driven by the state learning standards (see Ohio Learning Standards, 2011). Instruction engaged students in making connections among mathematical representations to help deepen understanding as proposed by NCTM (2014). Throughout instruction, students were asked to find multiple strategies for representing exponential growth functions. Students also were asked to justify why they used a certain strategy for problem solving. This idea came from Herman (2007) who suggested that when students are exposed to representations used by their peers they may implement them in the future.

The importance of this study is to provide insight into students' thought processes of why they choose certain representations. Investing time into understanding why students use specific representations allows educators to further understand how students approach problems, which aligns with Perry and Atkins (2002). This understanding can help inform instruction and allow educators to adjust instruction to fit the needs of the students.

Limitations

The first limitation of this study was that it was completed with only 22 eighth-grade students in one section. These students were not representative of the entire student population and thus the results may vary in other classrooms. Second, this study was conducted over a three-week period in which students explored exponential growth functions. Students had limited prior knowledge of exponential functions which could have played a role in lower number of correct answers on the pretest. A third limitation of this study was that different standards might result in different results from an identical study (see Ohio Learning Standards, 2011).

The unit on exponential growth functions was conducive for using multiple representations. This made it easier for students to find numerous representations and allowed students to share their representations with the rest of the class. However, there are certain mathematical concepts that do not necessarily lend themselves to employing multiple representations as easily as exponential functions. One example would be the eighth-grade geometry standards for transformations. Students explore the rules of transformations (see 8.G.A.3) but only work with showing a transformation by drawing the transformation on a coordinate grid and writing the rule. There is no expectation for working with transformations in a symbolic manner. The expectations drawn from the standards might limit students' representations. On the flip side, there are other concepts that would be useful for studying multiple representations. One example would be a unit on equivalent expressions, which is another eighth-grade standard (see 8.EE.C.7). A unit on equivalent expressions would be conducive for multiple representations because students could find solutions through graphs, tables, and equations. Also, students would have the opportunity to explore multiple equivalent expressions and draw diagrams to represent the expressions. This unit would allow the students to further explore multiple representations.

Future Investigation

If this study was to be repeated, then a larger sample might be appropriate to better represent the student population. Additionally, another topic should be investigated, possibly one that students are more familiar with. If students had the opportunity to explore a concept that was somewhat more familiar, then they may already know some representations, allowing them to investigate, which might leverage more representations during problem solving. A different concept could be more conducive to using multiple representations than exponential functions, again increasing the number of representations students employ. Finally, a longer study might give students the opportunity to further explore the given concept as well as understand the need for more representations. This additional time might allow students to become more familiar with new representations, which would increase their tendency to use those representations. More processing time would help students become more familiar with a new concept, which, again, could affect the number of representations utilized and the effectiveness of the representations.

Conclusion

The purpose of this study was to explore students' use of multiple representations in the classroom and their reasoning for representation choice. A pretest was given to identify representations used to represent exponential growth functions as well as students' understanding of exponential growth functions. Upon completion of the pretest, three students were interviewed to identify their use and justification of their chosen representation. A two-week unit was taught with a focus on using multiple representations. A posttest was given and a second set of interviews conducted,\ with the same three students to determine if there was an increase in the number of representations utilized and if students' justification for their representations changed. The results indicated that after instruction, both the number of representations utilized as well as the number of correct responses increased from the pretest. Additionally, students could more clearly elaborate on their justification for using certain representations. Students could also identify the benefits of using multiple representations after the two weeks of instruction.

References

Bostic, J. D., & Pape, S. J. (2010). Examining students' perceptions of two graphing technologies and their impact on problem solving. *Journal of Computers in Mathematics and Science Teaching, 29*, 139-154.

Bostic, J. D., Pape, S. J., Jacobbe, T. (2016). Encouraging sixth-grade students problem-solving performance by teaching through problem solving. *Investigations in Mathematics Learning, 8(3)*, 30-58.

Creswell, J. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Upper Saddle River, NJ: Pearson/Merrill Prentice Hall.

Goldin, G., & Kaput, J. (1996). A Joint Perspective on the Idea of Representation in Learning and Doing Mathematics. In *Theories of Mathematical Learning* (pp. 397-430). Mahwah, NJ: Lawrence Erlbaum Associates.

Heinze, A., Star, J., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM*, 535-540.

Herman, M. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *Journal of computers in mathematics and science teaching, 26*, 27-54.

Lesh, R., & Doerr, H. (2003). Foundations of a Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving. In *Beyond Constructivism: Models and Modeling Perspectives on Mathematics, Problem Solving, Learning, and Teaching*. (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum Associates.

National Council for Teachers of Mathematics. *Principles and standards for school mathematics*. (2000). Reston, VA: Author.

National Council for Teachers of Mathematics. *Principles to Actions: Ensuring mathematical success for all.* (2014). Reston, VA: Author.

Perry, J., & Atkins, S. (2002). It's not just notation: Valuing children's representations. *Teaching Children Mathematics, 9,* 196-201.

Piez, C., & Voxman, M. (1997). Multiple Representations: Using Different Perspectives to Form a Clearer Picture. *Mathematics Teacher, 90*(2), 164-167.

Preston, R., & Garner, A. (2003). Representation as a vehicle for solving and communicating. *Mathematics Teaching in the Middle School, 9,* 38-43.

Take Wolfram|Alpha anywhere... (n.d.). Retrieved from http://www.wolframalpha.com/input/?i=x^2+5x+4=0. Digital Image.

Tripathi, P. (2008). Developing Mathematical Understanding through Multiple Representations. *Mathematics Teaching in the Middle School, 13*, 438-444.

Von Glasersfeld, E. (1991). Abstraction, re-presentation, and reflection. In L. P. Steffe (Ed.), *Epistemological foundations of mathematical experience*. (pp. 45-67). New York, NY: Springer.

Yee, S., & Bostic, J. (2014). Developing a contextualization of students' mathematical problem solving. *Journal for Mathematical Behavior,36*, 1-19.

Appendix A

Pre-/Posttest

Name:_________________________________ Date:_____________ Hour: ____

1. Scientists are studying a human cells. They notice that a cell doubles every hour. This means if they start with one cell by hour one there are two cells, by hour two there are four cells. If this pattern continues how many cells will there be at hour 5? Show all of your work.
   a.
   b. Can you find the answer in a different way?
   c. Can you find the answer in a different way?

2. Suzie is starting to catch a cold and sneezes into her hand. Suzie forgot to wash her hand and meets up with her friends, Emily, Laura, and Leah, and shakes their hands, now they all have the cold. Emily, Laura, and Leah each meet up with three more friends, and pass the cold onto them. If this pattern continues for four more times, how many people will have the cold? Show all your work.
    a.

    b. Can you find the answer in a different way?

    c. Can you find the answer in a different way?

3. Molly is trying to save money to buy a new car. She makes a plan to save her money so she can buy a car. The first week she puts a penny in her savings jar, the second week she puts a nickel, the third week she puts in a quarter. If Molly continues this pattern, how much money does she need to put in her jar on week seven? Show all your work.
    a.

    b. Can you find the answer in a different way?

    c. Can you find the answer in a different way?