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W. Robert Midden
Bowling Green State University, midden.bgsu@gmail.com

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W. Robert Midden*
Chemistry Department, Bowling Green State University, Bowling Green, OH 43403-0213

A paper published in this *Journal* (*1*) proposed a "New System" for rounding numbers. It stated that the common practice of rounding numbers is flawed when 5 is rounded to even digits (i.e., when the digit being dropped is 5, the preceding digit is increased if it is odd and kept the same if it is even). The paper proposed that in rounding numbers, the digit preceding a dropped 5 should always be rounded up. It explains that in rounding the digits 0–9, it is more "equitable" to round five digits down (0–4) and five digits up (5–9) rather than to round five digits down (0–4), four digits up (6–9) and one digit (5) up half the time and down half the time. But some analysis reveals the flaw in this reasoning.

The problem arises from the error of rounding. If 5 is always rounded up, then the error introduced by rounding will accumulate. To recognize this, consider the error introduced in rounding each of the digits. There is no error introduced by rounding the digit 0 but there is error introduced by rounding any other digit. So the error from rounding four digits down (1–4) and five digits up (5–9) (rounding 0 introduces no error) accumulates as shown in the table. The errors are shown for rounding with the two different schemes ("Round 5 Up" refers to always rounding the digit preceding 5 up and "Round 5 Even" refers to rounding the digit preceding 5 up or down so that it is an even digit).

The errors accumulate with the "Round 5 Up" rule, whereas the errors sum to zero with the "Round 5 Even" rule. That's because with "Round 5 Up" there are no −5 errors to cancel the +5 errors that arise from rounding 5 up, but with "Round 5 Even" there are negative errors that cancel all positive errors.

An exception exists for numbers in which more than one significant digit is being dropped. In such a case, the other dropped digits must also be considered. This would apply, for instance, to the result of a calculation involving measured numbers. An example is:

$$35.7 \div 41.2 = 0.866504854$$

This result should be rounded to three significant figures and the digits following the first digit to be dropped (5) have some significance. Since the number following 5 is greater than zero, this number should be rounded up; that is, the "Round 5 Even" rule should not be used. That's because this number is actually closer to 0.867 than it is to 0.866; therefore rounding even in this case would introduce more error.

Errors Generated by Rounding Numbers 1–20

Number	Round 5 Up	Error	Round 5 Even	Error
1	0	−1	0	−1
2	0	−2	0	−2
3	0	−3	0	−3
4	0	−4	0	−4
5	10	+5	0	−5
6	10	+4	10	+4
7	10	+3	10	+3
8	10	+2	10	+2
9	10	+1	10	+1
10	10	0	10	0
11	10	−1	10	−1
12	10	−2	10	−2
13	10	−3	10	−3
14	10	−4	10	−4
15	20	+5	20	+5
16	20	+4	20	+4
17	20	+3	20	+3
18	20	+2	20	+2
19	20	+1	20	+1
20	20	0	20	0

Thus the new method previously proposed (*1*) should be used only for numbers in which the nonzero digits following a dropped 5 have some significance. Otherwise, errors are minimized for a uniform distribution of numbers by the traditional method of rounding digits 1–4 down and 6–9 up, and rounding 5 up half the time and down half the time. A consistent way to ensure this is to always round the digit preceding 5 to an even digit.

Acknowledgment

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Literature Cited

1. Guare, C. J. *J. Chem. Educ.* **1991,** *68*, 818.

*Voice: 419/372-6889; fax: 419/372-9809; email: *midden@bgnet.bgsu.edu.*