The Effects of Inquiry Problems on Students Construction of Mathematical Reasoning and Viable Arguments

Georgia Ike
Bowling Green State University, ikeg@bgsu.edu

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THE EFFECTS OF INQUIRY PROBLEMS ON STUDENTS CONSTRUCTION OF
MATHEMATICAL REASONING AND VIABLE ARGUMENTS

GEORGIA IKE

HONORS PROJECT

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_________________________________________________ Advisor
Dr. Gabriel Matney, School of Teaching and Learning

_________________________________________________ Advisor
Sandra Zirkes, Department of Mathematics & Statistics
The Effects of Inquiry Problems on Students Construction of Mathematical Reasoning and Viable Arguments

Abstract

This research study addressed the inquiry problem solving technique and its effects on students’ ability to construct viable arguments and critique the reasoning of others. The participants included six classes of secondary mathematics students. In order to answer the question “How do inquiry-based problems worked on outside of class time affect the students’ ability to construct viable arguments and critique the reasoning of others?” the research method involved noting how often different types of responses were used in class discussion, administering and evaluating an inquiry based assignment, observing students’ presentations and explanations of reasoning to the class, and interviewing students to compare their perspectives with observations made during class time. Findings suggest that students who employ the written and verbal explanations of their reasoning hold a better understanding of the mathematics at hand. However, many building blocks must be provided so students are able to successfully implement these practices.

Introduction

Having students create their own mathematical ideas is highly emphasized in the Common Core Standards. Upon attending the 2012 Ohio Council of Teachers of Mathematics Conference, I met a teacher who used problem-solving based problems to enhance the mathematical discussion in her 8th grade classroom. Her students worked on the problems outside of class, had to provide detailed work to support their solution, and then participated in a set of classroom presentations. During the presentations one or two students showed and explained their solutions to the class, students in the class were able to disagree and make arguments to debunk or help support the student’s solution. She showed a video that depicted a very successful outcome where students were deeply engaged in mathematical discourse and
formatting their notions and conclusions about mathematic concepts. This experience made me realize that incorporating problem-solving based learning and creating an environment and classroom situations where students are able to participate in thorough mathematical discussions is important for student ability to understand mathematics. I decided to research the topic further and integrate a similar experience in my student teaching classroom to see what benefits could come from this practice.

The Standards for Mathematical Practice (SMP) are a pivotal and vital component of the Common Core State Standards for Mathematics (CCSSI, 2010). According to the Common Core State Standards for Mathematics (CCSSM) the “Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important ‘processes and proficiencies’ with longstanding importance in mathematics education” (p. 6). Of the eight Standards for Practice, this research will be focusing on number three: Construct viable arguments and critique the reasoning of others. Students proficient in this standard will have the ability to justify their conclusions, reason inductively and deductively, and make plausible arguments within the context from which data arises. They will also be able to communicate their ideas and respond to the arguments of others, compare two plausible arguments, distinguish correct logic or reasoning, and explain flaws in arguments (CCSSI, 2010).

In this study we will observe the effect that inquiry-based learning that takes place outside of the class time has on the process of student learning when constructing viable arguments and critiquing the reasoning of others. Inquiry-based teaching and learning is a mathematical education practice that is on the rise in today’s classrooms. Everyone seems to be talking about it, but what exactly is it? Inquiry-based learning is a type of problem solving,
another term that is continually mentioned in the education field. Carjuzaa and Kellough define problem solving as “the ability to define or describe a problem, determine the desired outcome, select possible solutions, choose strategies, test trial solutions, evaluate outcomes, and revise these steps where necessary” (2012, p. 234). Inquiry is an avenue to develop problem solving in students. When using inquiry based strategies, learning occurs through the identification of a problem, the development of a hypothesis, the testing of the hypothesis, and the arrival at a conclusion (Carjuzaa & Kellough, 2012).

In the study described in this paper, I will be incorporating inquiry-based problem solving into my curriculum in order to discover what effect finding solutions, providing a written explanation, and presenting reasoning of inquiry problems will have on students’ abilities to construct viable arguments for their reasoning and critique the reasoning of others. These results will be important for mathematics teachers as the emphasis on the mathematics practice standards becomes more pressing. Finding ways to teach not only mathematics content, but also the Standards for Mathematical Practice is vital to helping students develop critical thinking, reasoning, communication, and critiquing skills that they will use in the mathematics classroom and in life outside of the classroom (NCTM, 2009). These essential skills must be established and flourished through a unique type of mathematical instruction that will be further investigated through this action research study.

**Literature Review**

Incorporating the inquiry-based learning strategy into the classroom can have many beneficial effects. Inquiry-based problem solving has been extensively studied and has been shown to have many benefits for learners, one of the most important and pertinent benefit is cognitive understanding. Fi and Degner (2012) begin their article *Teaching through Problem*
Solving with “Teaching through Problem Solving is an effective way to teach mathematics for understanding. It also provides students with a way to learn mathematics with understanding” (2012 p. 455). Inquiry and problem solving is focused on providing students with the opportunity to discover patterns and connections, use their reasoning skills, and formulate conclusions based on given data. When students make these connections themselves they are developing their understanding and are more motivated to comprehend how and why the process or solution works instead of just finding a quick answer (Schettino, 2011).

The process of inquiry learning actually contributes to gains in students’ ability to understand. In the process students are urged and often must use multiple representations in order to reason and sense-make with the information given and explain their reasoning to others. They quickly learn how to make connections themselves, demonstrate multiple connected and meaningful representations, understand and carry out the mathematical practice of problem solving, and communicate their ideas in various ways using the representations they have constructed (Fi & Degner, 2012); (Ali et. al., 2010); (Schettino, 2011). In addition to representing and communicating to enhance understanding, inquiry learning gives students empowerment in their voice of knowledge because they themselves have developed a solution to a complicated problem instead of just watching the teacher find the solution or working through a guided worksheet, where little thought is involved (Schettino, 2011).

Although the benefits are many and great, using this method is not always flowers and rainbows for students. The sole goal of inquiry-based problem solving is for students to struggle. John Dewey was a strong supporter of learning by doing and felt that a simplified math curriculum would deprive students of the learning opportunities to experience thought provoking character that led to the wisdom found in true human knowledge (Chen, 2012). Less is More is
an article that explains the importance of allowing students to discover concepts on their own given a very minimal amount of information and instruction. It gives two examples of teachers who provided too much guidance and help in order to make math “easier” for students, when in fact they were hindering student learning. By giving students too much of the answer or setting a lesson up so that connections are easily made, students will not be inquired to ask the questions that will help them to actually understand the mathematical concepts (Chen, 2012). Instead, they are just memorizing in a different way that looks like inquiry-based learning and accepting reasoning as fact without exploring and first-handedly learning in order to understand.

“The idea of letting each student struggle – productively – through mathematics and engage in mathematical practices is the meaning of the Equity Principle as outlined by NCTM (Fi & Degner, 2012). The Equity Principle communicates that, “All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study--and support to learn—mathematics” (NCTM, 2012). Many students are disadvantaged in some way and thus do not get the opportunity to learn mathematics as they should. Whether this is sole classroom instruction, or means provided outside of the school day or outside of the school itself, students are entitled to and should be provided these opportunities (NCTM, 2012).

Requiring students to “struggle” may not seem very equitable, but in fact the struggle, is what allows students to reason and make sense of ideas. Allowing them to struggle does not mean that no help is provided; it means that students are not just given answers or the means of finding the answer. They must put forth their own thinking abilities and use that to make connections, eventually leading them to a solution. Inquiry-based problem solving in the classroom reverses the simplified mathematics curriculum that has been so prevalent in mathematics classrooms in recent decades and requires students to think and reason for
themselves. A study completed by Riasat Ali, et. al. investigates problem-solving and its role in today’s classrooms. It discusses learning by doing, student centered learning and the positive role these concepts provide in a problem-solving based classroom. It also discusses the benefits of problem solving: information sharing with others, communication, the development of independent learning, and the opportunity to become a life-long learner when using problem-solving techniques. The study was conducted with seventy six 8th graders, half taught traditionally and half taught in a problem solving-based classroom format. It was determined that the students taught through the problem-solving based method had a better achievement rate than the students taught through the traditional method. One of these reasons was due to the development of independent learning (Ali et. al., 2010).

What does it mean for students to be independent learners? Independent learners are self-directed; they decide individually and consciously on the learning strategy and time scale they want to follow and carry the processes out by dictation of their own means (Ali et. al., 2010). A student has the opportunity to develop this independence through problem solving because they are working in situations where they are interested in the complex, thought provoking questions that are posed, while given the freedom to come up with their own reasoning and solving strategies. By allowing students to develop this independence in reasoning and problem solving, hand-in-hand, they are developing the tools to become life-long learners and successful individuals both mathematically and socially (NCTM, 2009).

Another important part of developing independent learners in the mathematics classroom is writing. Writing is not a prominent practice incorporated in the math curriculum. Although many students feel they are “bad at math,” the one thing that they do like about math class is that there is no writing. According to Ariana Stanca Vacaretu, her students were struggling with the
ability to read mathematical texts for problem-solving purposes. They could read very well, but they were having trouble with texts including mathematical definitions, explanations, or proofs, which led to the inability to represent a word problem describing real-life situations in a mathematical context (Vacaretu, 2008). To improve their problem solving skills, she first taught them to set up a problem in the format of an “If, then” statement, where the relevant information was placed with the If, as a premise, and the desired conclusion was placed with the Then. By then writing their own questions, critiquing peers questions, and revisiting their own problems, the students’ abilities to decipher and solve word problems improved (Vacaretu, 2008).

Another study was done by a sixth grade mathematics teacher in a similar situation, whose students were transitioning from elementary to middle school mathematics. She conducted action research and began by having students explain their reasoning of a problem-solving based question by writing their solution and explanation of the solution to a pre-service teacher pen-pal. The correspondence went back and forth and the pre-service teachers probed student thinking to advance student reasoning skills. The teacher discovered that students were having trouble writing their reasoning because they did not fully understand the underlying mathematic concepts, but as they continued to write and come up with questions, they became much more proficient (Lynch & Bolyard, 2012). Following this activity, she began implementing the *Mathematics Teaching in the Middle School* Palette of Problems while continuing to use the written explanations.

The teacher’s use of explicit self-monitoring questions and writing about problem solving in the classroom helped her students develop metacognitive and mathematical communication skills, both written and verbal. Written communication is especially important because, “Writing about problem solving gives students opportunities for reflection and creates a record that
captures and preserves students’ ideas” (Lynch & Bolyard, 2012). As students write and reflect on the reasoning skills they have used to reach a conclusion, they are further enhancing their understanding of the process they have completed. This written account also “creates a record of the students’ work, allowing educators to assess strengths and weaknesses in students’ metacognitive and mathematical problem-solving skills and then design instruction to target students’ needs” (Lynch & Bolyard, 2012). This is very beneficial, allowing teachers to know where students are in their understanding. Furthermore, they have data to use towards evaluating their teaching style and strategies for individual students and classrooms and to accommodate student needs accordingly.

In addition to writing in mathematics, communication amongst students and teachers about mathematics, in mathematics, and with mathematics is something that was found to be very important when conducting and analyzing the lesson titled, An Invitation to Try Pizza, Brenner (2002). An Invitation to Try Pizza brings informal mathematics representations into the classroom as a way to convey mathematic ideas in lessons. The research article addresses reform classrooms where the teacher structures instruction to include a wide variety of opportunities for students to discuss math. The communication, whether through small group work, class presentations, writing prompts, or teacher guided large group discussion, is essential when making connections between classroom math and math in everyday life, as well as allowing students to come up with their own means of finding solutions without a given formula or procedure. It was shown that making connections to real issues, such as buying the pizza that gives the most for your money, students’ everyday knowledge of mathematics could be used to enhance their understanding of new mathematics concepts (Brenner, 2002).
The previous case studies used certain processes or strategies to promote and provide scaffolding for inquiry learning. Another way to view the process of inquiry-based learning is through the development of specializing. “Specializing is an investigative process that serves as an entry point to a problem” (Lane, 2011). Specializing occurs when students consider specific examples of a problem. There are three types of specializing: random, systematic, and artful. Random specializing allows students to get a feel for the question by practicing specific examples. Systematic specializing is used to gain a more solid understanding and detect relationships and patterns among a larger number of examples. And artful specializing examines key concepts underlying a generalization. As a student becomes more knowledgeable of the problem through specializing, they are able to apply that knowledge to generalize a solution or conjecture that fits the entire data set being considered. Learning the specializing process will aid students’ capacity to problem solve and reason mathematically. It enables them to understand what a problem is asking and how that needs to be developed in order to find a solution (Lane, 2011).

Incorporating inquiry-based learning into today’s classrooms and curriculum sounds beneficial, but how probable is it when trying to align to the standards? The article Reflections on Teaching with a Standards-Based Curriculum: A Conversation Among Mathematics Educators, written by Newton, Geller, Umbeck, and Kasmer in 2012, studies a group of teachers who begin incorporating the use of Standards-Based Learning in their classroom. The teachers discuss the struggles that they had when trying to change their teaching habits to implement the Standards-Based Learning technique by using an inquiry-based model with a Launch-Explore-Summary structure. A veteran teacher experienced in teaching with a Standards-Based Curriculum and effective inquiry learning gives insightful answers to these questions based on the mathematical
discourse, the Summary portion of the lessons, and general pedagogy. The authors conclude that, “Using the Launch-Explore-Summary model or Standards-based practices is no easy task, particularly because this is not the way that most of us learned mathematics” (Newton et. al., 2012). However, they go on to say that using these teaching strategies will enhance student learning, engagement, comprehension, and overall knowledge. It will also move forward our journey in mathematics as we continue to develop and improve our practice as teachers (Newton et. al., 2012).

**Methodology**

In order to study the effect that inquiry problems have on the students’ overall ability to construct viable arguments for their reasoning and critique the reasoning of others in my student teaching classroom, I first observed class time during my mentor teacher’s teaching. I observed the occurrences that students were required to explain or provide arguments for their own reasoning and if and how adequately they were able to support their ideas and explain them to/defend them against other individuals who were not sold.

The *inquiry problems* are a set of problems obtained from the Palette of Problems published in the journal *Teaching Mathematics in the Middle School* founded by the National Council of Teachers of Mathematics. These problems are appropriate because they are created by NCTM to require reasoning in order to obtain a solution. Each is developed so that students with a wide array of capabilities can attempt and be successful through solutions found using various representations and strategies. A full list of problems and final solutions can be found in Appendix A and the assignment that was distributed to students can be found in Appendix B. The students were given seven of these problem-solving based problems for a five day period. The problems were of varying degrees to provide reachable challenges for all students. Students
were instructed to work on the problems outside of class time. They were labeled as homework and no other assignments were given for completion outside of class during that time. The students were aware that this activity was worth more points than normal homework assignments and that they would be talking about one of their solutions in front of the class. They were permitted to discuss ideas with other students, but had to submit their own work. Students were instructed to direct questions to me, the cooperating classroom teacher, family members, etc.

Each student was required to provide solutions to at least three of the given problems at the end of the allotted five day period. Solutions were graded on the student’s ability to provide detailed work to show their solution process and a thorough written explanation of the reasoning of their work. Additionally, at the end of the five days, each student chose the solution of one problem they completed to present to the class. They first explained their solution and reasoning, which was visible on the Smart Board through the Document Camera. Three to four students presented their reasoning for each solution. During that time I and other students asked questions to better understand the presenting student’s reasoning. The presenting student practiced providing an argument for their answer, considering other student’s questions, proposals or solutions, and comparing and contrasting these through discussion.

In order to promote open discussion during this time, I continually expressed to students that it was okay to have an incorrect solution. If they realized it before they presented, then they were encouraged to explain why their solution was wrong and what kind of thinking led them to the wrong solution, as well as a new strategy they could use to find a correct solution. In addition to promoting a positive atmosphere during the discussion, I have worked throughout my student teaching experience to create a safe environment where making an effort to think about math is placed at a higher value than presenting a correct answer. I have in no way chastised students for
giving wrong answers and oftentimes I am able to say things like, “I like the direction in which you are going, but I am looking for something more,” or “I like where you started, but let’s go in a different direction.”

Dialogue between students and the exchange of varying thought processes was strongly encouraged. Each student was observed and rated on their ability to explain the reasoning of the problem they presented and critique the reasoning of their classmates or reasoning I presented when those opportunities arose. I video recorded each class as I guided the presentations and discussion in order to go back and re-evaluate presentations if need be. At the conclusion of the presentations I asked each class for feedback on what students liked about the project and what they disliked. I called on all students who had ideas to add to the discussion.

After completing my teaching unit, I graded the inquiry problems based on my scaled rubric (Appendix C). I recorded the degree to which students explained their reasoning for each problem individually and for the assignment as a whole. Each student was graded on a five point scale for each of the three solutions they submitted and additionally for the presentation of their reasoning to the class. With these scores I conducted data analysis on the comparison of the presentation scores and the scores on the written work.

Finally, I conducted short student interviews (Appendix D) about the classroom discussion to take into account the students’ perception. Through the interviews I was able to gauge student feelings towards the degree to which they were able to provide information in class when working on these problems outside of class, the degree to which they now feel inclined to critique the reasoning of others, and the reasons they have these feelings. By doing this, I was able to determine if the students’ perceptions of this activity correlate with what I
observed through the written solutions and classroom discussions and to gain a better understanding on the effects that the activity had on various students.

These data sets and observations together will help me to gauge my students’ ability to explain their mathematical reasoning before, during, and after the inquiry problems and to gauge the effect the inquiry problem assignment had on student mathematical proficiency involving the construction of viable arguments and critiquing the reasoning of others.

Results

Pre-Assignment Observations

Through observations of my Mentor Teacher’s teaching, I found that she uses a combination of teacher-centered, direct instruction and student-centered, problem-solving based learning. My mentor teacher formulated questions that required the students to think critically about a topic, and produce viable arguments laced with reasoning; however students did not often build on peers reasoning or point out disagreements with peer reasoning. On many occasions they just said what they wanted to say, provided reasoning, but did not take previously discussed solutions into account in their own explanation.

I also found that as I took over the main educator role in the classroom, these habits did not change. I was able to ask questions that provoked students to think critically and explain their solutions with strong reasoning support, but students did not build on peer explanations or use their own thinking to debunk another individual’s reasoning.

Post-Assignment Student Opinions

After conducting the in-class presentations I asked each class how they felt about the project, what types of things they liked, and what aspects they disliked. Some of the common responses that students provided included:
• “I like the genius problems more than the book problems.” (Like)

• “They are more challenging.” “It was difficult.” (Like and Dislike - When asked if this is a good or bad thing, many of the students said it was good.)

• “You could pick the three you wanted to do.” (Like)

• “It is more fun than doing straight up math.” “It had different varieties of questions like the farmer thing that was just a word question and it wasn’t all just math.” (Like)

• “Less questions than usual and more time to work on them.” (Like)

Some less-than common, but still very interesting responses that students provided included:

• “I said I didn’t want to go up, but when I did I was like this is not as tough as I thought it was going to be.” (Like)

• “I didn’t like how the questions were confusing.” (Dislike)

• “I like that we got to present and it got quiet people to speak.” (Like)

• “It’s different strategies for each problem. I could choose what I wanted to do [to solve] the problem.” “There were some different answers and different ways to find out the answers.” (Like)

• “Shows what they [each student] did wrong and how they corrected it.” “See what the real answers were and see how to do it and see what other people did and compare your stuff.” (Like)

• “Since you have to go up there and present … later on when you have a bigger project you don’t get a bad grade because .. [you have that experience].” (Like)

• “I think it challenged my brain. (Is that a good thing?) Yeah. It helped us with our speech skills.” (Like)
When reviewing all comments that students shared following the presentations, about two out of every three comments from students were positive. Many of the negative comments stemmed from the second bullet above when students said that the problems were too difficult or confusing.

**Classroom Presentations**

I assigned this activity to all six classes of seventh grade students that I teach, totaling 137 students. This includes students of all ability levels, high to low, and a total of fourteen students who hold IEPs or 504 Plans. There was a mixture of effort that was put towards the activity by students. As mentioned earlier, the presentations were graded on a scale of 0 to 5, with 0 being no effort and 5 including a well thought explanation with reasoning to support the solution. Some students took it very seriously and provided well-thought explanations and responses in their presentation. These students either scored 5 out of 5 (45 students - 32.8%) by providing a very detailed explanation and justification for their solution or they scored a 4 out of 5 (34 students - 24.8%) by providing an adequate explanation and justification for their solution, but forgetting to mention some important details that contributed to the reasoning of their solution process. These top two scores comprised of almost 57% of all of the students.

Other students put forth effort, but had problems adequately explaining their solutions, omitted very important details for the reasoning process, and got confused or lost when trying to answer questions from me and their peers. These students were divided evenly between scores of 2 (16 students - 11.7%) and 3 (16 students - 11.7%). This shows that just under one-fourth of the students, after putting forth some effort, had trouble explaining and justifying their reasoning and struggled when attempting to think about alternative approaches or about their own approach more deeply.
The last group of students either did not try and thus did not present, or put a very minimal effort towards their solutions and gave inadequate explanations, could not explain their work, or said things like “I googled it,” and had no idea why their answer was correct (or incorrect). Three of the 137 students (2%) scored a 1 out of 5 with the minimal effort and explanation that was given, while 23 students (16.8%) scored a zero out of 5. Of those 23 students, 16 students did not attempt to present one of their solutions. Additionally, of the 23 students, 8 did not attempt the assignment. More details of what comprised each score can be found on the scoring rubric (Appendix C).

The mean presentation score of all 137 students was a 3.24 out of 5 points. According to the rubric, this shows an average ability across all classes of students to explain reasoning and give justification for reasoning. Although this includes the zero scores of the students who did not attempt the problems, the presentation, or both in the cases of those eight students mentioned earlier. When excluding the eight students who did not attempt the assignment at all, the mean presentation score for these 129 students became 3.44 out of 5. Additionally, when excluding the 16 students who did not attempt to present a solution, the mean presentation score for these 121 students became 3.67 out of 5.

During the presentations, I noticed an interesting trend. As the students presented during class, and lacked a very detailed explanation from the start, I asked many of them what their plan for solving was. A common response to that question was that they did not have a plan. Sometimes they would say they guessed, which they had obviously not done, or they would say they just knew it, which was very seldom the case. After a little more coaxing or a more specified question, most students proceeded to tell me a strategy or some type of tactic they used to begin their thinking and solution process. I realized as the activity was progressing that many
of the students did not realize they had formulated a plan in the first place, which created a struggle when trying to communicate their plan or strategy for solving. Implications of this will be discussed further in the conclusion of this paper.

**Written Solutions and Explanations**

The three solutions that each student submitted yielded interesting results. As mentioned earlier, I assigned this activity to all six of my 7th grade classes. The assignment sheet that each student received, including the 7 questions that were provided for the choosing, is attached as Appendix B. Additionally, the scoring material can be found in Appendix C under the second rubric titled Genius Problem Written Solution Score Rubric. Before discussing the details of the written work and explanations of reasoning, I want to provide a quick overview of the mean scores and patterns that existed in the solutions. Across all students who completed each solution, the mean scores were:

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Presentation</th>
<th>All Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Score</td>
<td>3.41</td>
<td>3.36</td>
<td>3.11</td>
<td>2.59</td>
<td>2.57</td>
<td>3.11</td>
<td>2.73</td>
<td>3.24</td>
<td>3.06</td>
</tr>
</tbody>
</table>

The mean scores of each of the seven solutions fell between 2.5 and 3.5 out of 5 points. These individual problem scores and the mean value of all of the problems (All Problems) take into account only the problems that were attempted, so they do not reflect students who did not complete the assignment, only students who attempted the question regardless of their score. The presentation score includes the zero and low scores of those students who did not participate or fully complete the assignment. These differences will be further analyzed shortly.

One aspect that was an integral part of this research study was the written explanations that students provided about their solution process and the reasoning behind their solutions.
Many of the students who participated did not give a written explanation. On the other hand, many showed their mathematical work and felt that showing work was enough of an explanation for their solution. However, these explanations and justifications of why the work was correct were a very important component of the research study. Many students shy away from writing in mathematics, but this explanation is important because a student can show all of their work without completely understanding why the work was appropriate and what question their solution is answering. This written explanation makes students aware of the knowledge they have and the process they are using to solve because they have to consider more deeply their thinking process. The lack of written explanation lowered the scores and deterred from what was being measured in the study.

One thing that was surprising after seeing these results was the many detailed and adequate explanations that students gave during class, in opposition to the lack of written explanation. The students seemed more willing to provide a verbal explanation during class time, than give the effort to write that explanation. Because there were so many students who did not complete the written explanation, I began to compare the scores of the students who completed the explanations with the scores of the students who did not complete the explanations. With 44 out of 137 students completing explanations for their solutions, accounting for just under one-third (32.12%) of the students, this analysis showed many interesting results. Of the students who completed both components, showing work and explaining reasoning, one student scored 4 out of 20, five students scored 9 out of 20, and the remaining students scored a 12 or above on the overall assignment. The table below shows the differences between the two categories.
The red represents the mean scores of all students who did not give explanations, including the students who did not do the assignment and received a zero. The orange represents these scores with the zero scores excluded. The mean scores show a rise with these scores eliminated. There is a much more noticeable rise in the mean scores when explanations were provided (green). The last two columns show the percent increase of the scores from No Explanation to Explanation Provided. If we focus on the students who completed the assignment, there is a substantial increase of almost 25% on the presentation scores and almost 33% on the total score on the assignment. Because the explanation piece was factored into the total score, a direct causation for the low scores exists. Therefore, the presentation scores are more conclusive because they specifically show that the students who did not support their work with a written reasoning scored lower on the presentations.

**Student Interviews**

In addition to observing the students presentations, grading their papers, and receiving feedback from the entire class on the activity, I conducted interviews with three students. The purpose of these interviews was to gain more insight as to what extent this activity affected students’ ability to make viable mathematical arguments and critique the reasoning of their peers.
Additionally, this provided feedback on the perceptions these students held of those processes taking place throughout the classroom before, during, and after the activity. Summaries of the interviews follow. Each student was given a pseudonym to protect their identity and privacy.

Each student was asked the following questions: (1) Describe your contribution to the mathematical classroom discussion that took place based on the work of these problems outside of class. (2) Compare your discussion on these problems with your normal classroom discussion. (3) How does having to write out your explanation affect your understanding of the solution(s) you found to the problems? (4) Did you find that you could make a viable argument and critique the reasoning of others after the activity? Why or why not?

Tori is an A average student. She normally contributes to the classroom discussion with very unique and thoughtful mathematic ideas. When I am urging students to extend their thinking to a higher level concept or explanation, Tori normally has already come to the specific solution by those means, and if not, she can quickly pick up on these higher-level strategies. She can also successfully provide reasoning to her solutions in almost all cases.

As the interview began, Tori discussed her classroom presentation and again provided detail about the steps she used and the original explanation she gave to her peers during her classroom presentation. When comparing her discussion on these problems with her normal discussion during class she quickly pointed out that she is normally very quiet in class so she usually only “talks when there is something that [she] disagrees with.” However, with this experience she had to present her initial solution and thinking instead of just providing an argument against a peer’s reasoning. Tori also discussed that she did not provide many contributions when she disagreed during the in-class presentations. Additionally, she experienced moments of confusion during other student’s presentations, but as the discussions progressed the
solutions and reasoning became clearer to her. In regards to the normal classroom experience, she thought that this experience was less chaotic (students were less talkative) than during normal class time because the focus was on one student at the front of the classroom.

When asked about the effect that writing out her solutions had on her understanding she said, “All the work, I usually do that, and that usually helps me out. Having to write out word-wise how I figured it out, that confused me a little bit. When I have to write out huge explanations [that confuses me], but when I just have to write out work, then I’m good.” That was a very interesting thing to hear from Tori because she usually seems to understand a wide array of solution strategies and can verbally explain them. In response to question four, which specifically asks about her ability during the activity to create viable arguments and critique the reasoning of others, the third Mathematical Practice Standard, Tori said, “I don’t really know, because it’s always easy for me.” Critiquing peer’s arguments is a little harder because she gets confused as she tries to follow their reasoning. When she heard clear explanations it was easier to think about the problems in different ways that other students may have used. Overall, she enjoyed arguing with her peers, but did not enjoy the extra effort it took to understand her peers’ thinking.

Mark was the second student that was interviewed. Mark is a B average student and is normally on task in math class. He can normally provide thought provoking responses and ideas during class discussion, but lacks in giving these types of responses in written work. His contribution to the class discussion revolved around question #6, with the farmer, goat, wolf, and cabbage. In the interview he talked about how he messed up in his solution, describing that his original idea to take the wolf over first would not work because the goat would eat the cabbage. He realized this mistake in class when peers were presenting their solutions to the problem.
During the interview he proceeded to describe the correct solution with detailed reasoning and understanding.

When comparing the presentations with normal classroom discussion, Mark, like Tori, called attention to the minimization of distractions during the presentations. He associated this to students wanting to explain their solutions without being distracted and being excited to use the Smart Board and show their own paper to the whole class. Whereas during a normal class time, students are sharing from their seats so the attention is not directly on them while they are sharing. He stressed that normally students are more prone to say, “That’s wrong!” but because I instructed the class to listen to the entire explanation before interjecting, there were less outbursts of noise.

Writing out the explanations to justify each solution was one aspect in which Mark said he lacked because he was able to just write out the work to get his solution. He thought it was easier to explain the Farmer problem because the main way to show work was, “to write out how you did it.” Mark said that if he would have focused more on writing an explanation for each problem, it would have made the problem harder to do, but easier to understand. He would rather explain his reasoning verbally than in writing. He felt that he was able to critique the reasoning of others during the presentations, especially in cases where the reasoning was obviously wrong. Mark discusses a case where a classmate did not follow the rules of the problem and he had to argue with the student to explain that the solution defied the parameters given. He was not sure if he is better able to critique the reasoning of others following the presentations. Overall, Mark felt like he better understood the reasoning behind each solution as a result of the structured presentations because he was able to see the exact work and reasoning of other students.
Luke is the final student who was interviewed. Luke is an A average student who is very talkative during class time. He can become disruptive to the entire class on occasion, but is very bright when it comes to mathematics. His contribution to the class discussion revolved around problem number seven. He gave me a brief explanation of his process for solving and his reasoning, struggled with this at first, but was able to articulate his ideas after talking through the process. When referring to the classroom environment, he points out that students rarely go to the board and that he “sometimes” explains his thinking during normal class discussion and on homework.

Showing his work and having to write about the process of finding the correct solution helped Luke to find patterns to help solve and determine his if his solutions were correct. He addressed that when he explained his solutions he was able to catch mistakes as he looked over his work to formulate an explanation. Luke’s interview was very interesting because he focused on the fact that showing his work made aware various patterns in the solution (# 7) and providing explanations helped him to find mistakes that he had made in his work (#1) as he revisited the work to formulate his explanation.

Luke felt that it was easy to critique the reasoning of other students in some cases when they explained their work well, but felt it was also hard because “you had to think a little bit.” He expressed that showing all of his work helped him understand these problems and formulating explanations has extended positively to the reasoning that he must provide in class and on homework following the experience. Overall, Luke felt that this was beneficial because it brought out explanations from those students who normally do not volunteer to share and he enjoyed finding arguments to disprove the arguments of peers. He believed that these arguments
from peers during the presentations helped him and other students to better understand the strategies and solutions.

**Conclusions and Implications**

As mentioned earlier, Tori is normally a very high level mathematical thinker. I was very surprised in her interview when she said that she was confused during the presentations and when trying to explain her work in writing. One reason that may have contributed to the former was that not all explanations were correct and some students whizzed through them without providing detailed reasoning. When discussing this, she specifically referred to the problem with the farmer, goat, wolf, and cabbage. She did not complete this problem before class time, which could have contributed to some of this confusion. Also, it was more logically based than mathematically based, which seemed to make explanations harder to follow during class time. Another contributing factor could have included the normal procedures during class. Students are given problems on homework that require explanations on occasion and are asked to verbally support their reasoning in class, but this does not provide students to gain the understanding that is so greatly associated with writing about solutions by Vacaretu (2008, p. 451-455), and Lynch and Bolyard (2012, p. 486-492) which was highlighted in the Literature Review.

Mark had similar things to say in regards to the environment of the classroom and the difficulty of writing out reasoning and justifications. I found this very interesting because one thing that I felt would be enhanced was the classroom discussion. Contrary to my supposition, the classroom was much more quiet and structured than normal. This isn’t to say that mathematical discourse was absent, but that a large amount of off-task distractive behavior was greatly minimized, so the mathematical discourse that did occur was more focused and intent. Mark also said that providing a written explanation would have made the process more difficult,
but would have increased his understanding. In Mark’s case, his original incorrect solution may have been caught before class time if he would have engaged in thinking about a written justification. This seemed to be the case for the majority of the students, who realized their mistake as class discussion began.

Another interesting thought that arose in our conversation was his preference to give a verbal over a written explanation. Again, this goes back to the two articles discussed in the literature review by Vacaretu (2008), and Lynch and Bolyard (2012) which focus on the benefits provided to students when putting mathematical discourse and problem-solving into writing. Normally verbal explanations are valued in a mathematics classroom, as they should be. However, vocabulary and underlying reasoning can often be assumed in a verbal explanation, so the full reasoning and explanation is not always being articulated. Because students are not used to communicating the details, the written explanation becomes much more difficult. Students need exposure to and practice with writing about mathematics and their reasoning as a means of understanding mathematics at a deeper level.

The conversation with Luke differed because he had completed the written explanations of his reasoning in the work that he submitted. He was able to evaluate how explaining his reasoning on the written portion actually affected him, not just how he thought it might help him. One key idea that he addressed was his ability to understand the solution through the work that he completed, in addition to the written reasoning he provided to support his work. He discussed a solution that he discovered was incorrect as he was communicating his reasoning through examining his solution. Going back to write his reasoning proved successful in his comprehension and deeper-level understanding of the problems at hand.
Each of these students added valuable observations to the effects, successes, and limitations of the research study. It was hard for them at points to talk about their reasoning because they had not participated in the written explanations that they were supposed to have provided for each solution. I realized after their comments that many of the class discussions were very one-sided with the student talking at the front of the classroom and myself or other students asking questions on occasion. Also, because many students were not prepared, more confusion about solutions may have been created. This is especially a red flag because the top-level and above average thinkers are conveying this message. From these interviews, it can be concluded that the three individual students felt they were either unaffected or affected positively in their ability to create viable arguments and critique the reasoning of others. None felt that they were less able to complete these two task and some felt much more confident in that ability.

When referring to the results discussed earlier, many ideas can be concluded from the data analysis. The students who expressed their explanations performed much better on the written and verbal presentations than the students who lacked explanations to show their reasoning. This probably is not caused directly by the explanations provided (or lack-there-of), but the correlation is important. Those students who successfully showed reasoning possessed much higher performance scores, which shows that this ability is important for understanding of the mathematical ideas. Furthermore, more accurate results would have been yielded if all students had at least tried the problems and written expression of reasoning. This includes the students’ ability to solve the problem, but especially their ability to create viable arguments and critique the reasoning of others.
Limitations

Through this research study there were limitations within the classroom environment that affected and may have inhibited the results that were found. First, the brevity of the study meant that students were only able to participate in the experience one time. This makes it difficult to determine the activity as a contributing factor to one specific outcome. With such a short time of study, many factors could have contributed to a positive or negative result. As an isolated activity that differed from the normal procedures in the classroom, it cannot be determined what the benefit might be if a teacher employed this process over the course of a semester or year. Also, because the problems and majority of the mathematical thinking was completed outside of class time, results cannot be determined if similar results would follow if this was used on a normal basis inside of the classroom.

Another limitation arises as a result of a handful of the students not completing the challenge problems, not presenting their solutions, or providing minimal effort to the activity. By not engaging in the activity, students with overall scores of zero and lower end scores decreased the group average in the data analysis of the final scores. This did not give an accurate portrayal of the collective effects on the students who participated. For example, in the Results section of the paper, the average scores for the presentations were discussed. The original mean was 3.24 out of 5. This increased to 3.44 out of 5 as the students who did not attempt the assignment at all were excluded and to 3.67 out of 5 as the students who did not present were eliminated from the data set. Because of the pattern of increase in these presentation mean values, it can be seen that the students who put effort towards the activity, did an average job of explaining their reasoning, justifying their solutions, and answering peer and teacher questions in class.
This lack of participation made it difficult to evaluate each student’s ability to explain their reasoning and critique the reasoning of others. As mentioned earlier there were eight students who did not attempt the assignment, and an additional eight who did not have enough work shown to begin to explain what they did, therefore did not participate in the presentation. The 23 total students who received a zero in the presentation column held a mean score of 3.87 out of 20 points. I believe that one of the contributing factors to this poor work was the very weak work ethic inside and outside of class. Many of these students struggle to complete their homework and, on occasion, refuse to participate in class. Because these were more challenging problems, they may have just decided not to complete the assignment or they may have tried, become quickly discouraged, and quit. The questions could have been overwhelming for the students who have low mathematical abilities, which also may have contributed to a lack of effort and participation.

In addition to a lack of participation, it was difficult to interview students who normally have lower math scores because those were many of the students who did not complete the activity at all or completed it with minimal effort and would not have provided thoughtful feedback. This was evident in the informal likes and dislikes that each class mentioned after the presentations. Many of the students who did not put forth effort in their solutions and presentations were the students who said they did not like the activity because it was too difficult or challenging, and these were also the students who scored very low across the board. The three students I was able to interview had grade averages as follows: 2 A’s and 1 B. The feedback from these three students was very interesting because the student with a B average gave much more thoughtful and well-articulated responses than the two A average students. It seemed that because he was not always used to having to provide well-thought solutions, he learned more
from the process and was more affected by providing his reasoning and listening to the reasoning that other students provided.

Finally, because there were only seven total problems and 20-25 students in each class, each question was discussed by at least three students. With multiple students giving their explanations, often times they would critique their own answer and express why it was wrong before other students were given the opportunity to verbally critique their peers. This was beneficial for the student presenting and the other students in the classroom, but it minimized the mathematical discourse that occurred during class time between students and the various types of critiques or arguments that would have arisen.

**For Future Teaching and Research**

In a future study and in my future teaching I would begin by creating a longer experience of similar structure. A quarter, semester, or even year-long study to better gauge the effects of using these types of questions for out-of-class activities would provide more substantial feedback than just one, isolated experience. Analyzing the evolving results that appear when doing these types of activities on a regular basis would be helpful to determining if these inquiry-based problems will be of benefit to student understanding and mathematical reasoning in the long run as many of the sources in the literature review supported. Creating and incorporating lessons and activities that take place during class time where this type of problem solving is stressed is another extension that I would implement in a future classroom. Establishing an environment where students are engaging in problem-solving and mathematical discourse in this way would deliver a better-rounded basis of the types of effects problem solving and inquiry based learning would have in the classroom.
My current group of students has not had much practice formulating plans and strategies to find solutions and using the reasoning principles that go along with this type of problem solving. It became very prevalent as the presentations ensued and as I viewed their written solutions that many did not know how to articulate their thinking and strategy of solving. Because of this, they seem to be very unaware of the types of strategies they consistently use and what it means to use a strategy or develop a plan for solving. Even the students that were able to effectively develop a strategy struggled with communicating that strategy to another individual.

In future teaching and research, I plan to focus on students’ abilities to communicate about math, develop strategies, and correctly express those strategies at the very beginning of the school year. The more experience students have discussing mathematics, the better they will be able to understand math and engage in fruitful mathematical discourse.

Additionally, when I begin implementing these types of activities that focus on problem-solving based learning, I will begin with work inside of the classroom. It was difficult to understand what the student was thinking in instances through their written work and even in their presentations. Focusing on the strategies mentioned above dealing with the solution process will provide students for a much better basis of how they might approach problems. Working on this in class will allow me to see what students are doing, what they know, and where there may be missing mathematical or logical pieces. This also promotes students to engage in a slightly different kind of mathematical discussion as they work together to find solutions. As they hash out ideas and strategies, valuable discussion and critiquing of peer’s arguments is likely to occur. Another change that I might make regards the presentations. If students work in class and have group discussions, easing them into presentations by allowing them to speak as a group or pairs
may be more beneficial until they become comfortable with expressing their ideas and explaining their reasoning to the class.
Bibliography


