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(Re)Considering Teachers' Promotion of the Standards for Mathematical Practice

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Please Note:

RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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(RE)CONSIDERING TEACHERS' PROMOTION OF THE STANDARDS FOR MATHEMATICAL PRACTICE

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This study investigated mathematics teachers' teaching practices and the ways they promoted the Standards for Mathematical Practice (SMPs) before and after yearlong professional development (PD). Our research questions are: (1) To what degree did teachers' promotion of the SMPs change after yearlong PD focused in this area? (2) Were there any differences between cohorts and/or grade-bands in their promotion of the SMPs? Results express that teachers' promotion of the SMPs grew significantly during the PD and there were significant differences between elementary and secondary teachers.

Although professional development (PD) and the National Council of Teachers of Mathematics ([NCTM]; 2000) process standards have been researched for several decades, the adoption of the Common Core State Standards for Mathematics (CCSSM) and development of the Standards for Mathematical Practice (SMPs) provide a new and important context for study. It is not a foregone conclusion that the existence of mathematical practices such as the SMPs necessarily implies that teachers promote them during instruction (Bostic & Matney, 2014a; Hiebert et al., 2005). Moreover, it cannot be implied that teachers have made sense of them (Bostic & Matney, 2014b; Olson, Olson, & Capen, 2014). The present study provides evidence of the effects of yearlong PD on teachers' instruction, particularly in their promotion of the SMPs, which are central to doing and learning classroom mathematics (CCSSI, 2010; Koestler, Felton, Bieda, & Otten, 2013). It builds from past research (e.g., Bostic & Matney, 2014a) with inclusion of new data from two similar PD programs. Furthermore, the study provides research-based implications for mathematics educators who provide PD to teachers. Undergirding the present study are two sets of literatures: research on teachers implementing the SMPs and research on professional development.

Related Literature

Standards for Mathematical Practice

The SMPs are part of the Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010). They offer characterizations of behaviors and habits that students should demonstrate while learning mathematics (Bostic &

Matney, 2016; CCSSI, 2010). The *Principles and Standards for School Mathematics* (NCTM, 2000) and *Adding it Up* (Kilpatrick, Swafford, & Findell, 2001) guided the descriptions of the SMPs. The literature is clear that teachers' instructional emphasis of the process standards, which promoted students' mathematical proficiency prior to the CCSSM, did not occur often (Hiebert et al., 2005). Initial research reports about CCSSM implementation suggests that K-12 teachers are struggling to make sense of the SMPs (Bostic & Matney, 2014b; Olson et al., 2014) much less weave the SMPs into their everyday instruction on the SMCs (Bostic & Matney, 2016). These findings suggest a need for research about professional development that enhances teachers' understanding of the SMPs and supports them to design and actualize instruction that makes the SMPs a part of their mathematics teaching.

Professional Development

We drew upon Guskey & Yoon's (2009) analysis of research about what works in PD when considering the design of the PD involved in this study. We sought to structure PD that adhered to the key features they found to be effective: (a) PD should have workshops focused on "research-based instructional practices" (p. 496) involving active-learning experiences for participants; (b) PD activities ought to encourage teachers to adapt a variety of practices to a specific content area, (c) PD should include a sufficient amount of time for teachers to make sense of the ideas and promote the application of these ideas during teachers' instruction; (d) PD ought to be structured and have sustained follow-up. A content-focused PD experience provided a space for teachers to apply a variety of practices to their classroom instruction. We utilized these features in tandem with the research-based work of NCTM's (2007) implementation standards for teaching and learning to provide teachers a conceptualization of teaching as sufficiently complex enough to promote student learning. The NCTM (2007) implementation standards define and emphasize the importance of worthwhile mathematical tasks, learning environment, and discourse. Past research has utilized these standards in PD. Boston (2012) detailed how focusing on implementing worthwhile tasks during a yearlong PD enhanced secondary teachers' knowledge, which in turn influenced their instructional practices. For example, after the yearlong PD they were able to identify elements of tasks with high cognitive demand and concurrently selected more tasks with high cognitive demand for their own instruction. Improving teachers' ability to select worthwhile tasks is not the only way to impact their instructional outcomes (Boston & Smith, 2009); supporting them to establish an effective learning environment and sustain mathematical discourse between students are also necessary to maximize students' opportunities to learn (NCTM, 2007). Our research builds upon Boston and others' work by adding a new layer into PD, the SMPs. The research questions for the present study are: (1) To what degree did teachers' promotion of the SMPs change after sustained (i.e., 100 or more hours) PD focused in this area? (2) Were there any differences between cohorts and/or grade-bands in their promotion of the SMPs?

Method

Context of the Professional Development

We aim to explore how teachers' instruction changed to promote the SMPs and connect this growth to PD projects. We focus on elementary and secondary teachers' experiences as influenced by four sets of teachers in sustained grant-funded professional development programs from one Midwest state. Three of those sets included cohorts of K-5 and grades 6-10 (i.e., Algebra 2) mathematics teachers who convened for a one-year program during 2012-2013, 2013-2014, and 2014-2015. These cohorts met for 100 hours during one calendar year. The fourth set included cohorts of K-5 and grades 6-8 mathematics teachers who convened for a two-year program (2014-2016) for a total of 256 PD hours (128 hours per year). For ease of reading, we name the set of K-5 and 6-8 cohorts from 2014-2016 "Apple" and "Blueberry" and the three sets of K-5 and 6-10 cohorts from 2012-2015 "Cherry". Generally speaking, the aim of the PD projects included (1) making sense of the SMPs, (2) exploring inquiry through NCTM's (2007) standards (i.e., worthwhile tasks, mathematical discourse, and appropriate learning environments), (3) implementing classroom-based tasks that aligned with the CCSSM, and (4) increasing mathematical knowledge and understanding. Teachers read and discussed chapters from NCTM books (e.g., *Mathematics Teaching Today* [NCTM, 2007]) and completed various assignments including journaling, writing, enacting, and reflecting on CCSSM-aligned mathematics lessons, and solving rich mathematics tasks. They also reflected on their mathematics instruction as well as the instruction of others implementing the CCSSM. Additionally, Apple and Blueberry cohorts engaged in lesson studies each semester, which were conducted at schools of participating teachers, while Cherry cohorts did not. Thus, the PD formats were fairly similar except for lesson study and number of hours met.

Participants

A total of 152 teachers participated in this study between the three PD programs. Table one shows teacher sample data by program they participated in and grade level they taught at the time of participation. Across the Apple cohort, 20 secondary teachers were part of the program. Thirty-four teachers composed the Blueberry cohort, (i.e., n= 23 elementary and n = 11 secondary). Ninety-four teachers participated in the Cherry cohorts; (n = 64 elementary and n= 35 secondary mathematics). Teachers for Apple, Blueberry, and Cherry cohorts came from urban, suburban, and rural school districts. All cohorts followed the same meeting format, used the same framework for the PD, but met in different parts of the Midwest state due to geographical constraints.

Table 1

Demographic Data for Teacher Participants

Demographic Variables	Frequency (%)
Program	
Apple	20 (13%)
Blueberry	34 (22%)
Cherry	98 (65%)
Grade Level	
Elementary	87 (57%)
Secondary	65 (43%)
Program x Grade Level	
Program A Elementary	0 (0%)
Program A Secondary	20 (100%)
Program B Elementary	23 (68%)
Program B Secondary	11 (32%)
Program C Elementary	64 (65%)
Program C Secondary	34 (35%)

Data Collection and Analysis

Teachers were asked to design, enact, and videotape one lesson when the PD began (i.e., pre-PD) and again near the end of the PD. For Cherry cohorts, this occurred after one year of PD

(100 hours), for Apple and Blueberry cohorts this occurred after two years of PD (256 hours). Depending on the grade level and the local school context of the teacher, the videos were as short as 25 minutes and as long as 65 minutes. Since our study focused on ways that teachers supported students' engagement in the SMPs during instruction, we investigated the videotapes as a means to report instructional changes made during the PD program. Such analysis approaches have been used in similar studies such as Boston (2012) and Boston and Smith (2009).

Data analysis required two parts. The first part involved watching the videotapes and reflecting on instruction using a protocol focused on the ways that teachers' instruction promoted the SMPs. Two mathematics education faculty as well as seven mathematics education graduate students watched the videotapes and conducted the analysis using a protocol validated for this purpose (see Bostic & Matney, 2016; Bostic, Matney, & Sondergeld, 2017). It provides look-fors that link mathematics teaching behaviors and the SMPs. For example, three aspects for the first SMP: Make sense of problems and persevere in solving them (CCSSI, 2010), include looking for the ways teachers (a) Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution, (b) Provide opportunities for students to solve problems that must have multiple solutions and/or strategies, and (c) Encourage students to represent their thinking while problem solving (Bostic & Matney, 2016). While there may be other aspects indicative of SMPs, the protocol provides an evidence-based framework for examining mathematics instruction using the SMP lens. Next, pairs of coders compared their observations with one another to gather interrater agreement. To maintain fidelity with use of the protocol, the team conducted meetings every few months with the sole purpose of establishing interrater agreement. The minimum threshold for interrater agreement is r_{wg} = .9 (James, Demaree, & Wolf, 1993). Interrater agreement exceeded the minimum threshold; it was as low as r_{wg} =.92 and as high as r_{wg} = 1.0]. Thus, we felt confident that our team was applying codes in a consistent manner.

The second part of data analysis focused on quantifying changes in the number and type of instructional opportunities related to the SMPs. The type and frequency of instructional opportunities related to each SMP were categorized. We then summed the pre-PD number of indicators for each SMP to create a grand total across all eight SMPs. The pre-PD grand total was compared to the post-PD grand total. Next, we completed a 2 Within, 2 X 3 Between

Factorial ANOVA to answer our research questions. The within independent variable was time (pre-PD and post-PD). Between independent variables were grade level taught (elementary or secondary) and PD program (Program Apple, Blueberry, or Cherry). The dependent variable was SMP score across all analyses.

Results

For RQ1, regardless of PD group or grade level taught, on average teachers expressed significantly more opportunities to promote the SMPs during post-PD instructional observations compared to pre-PD instructional observations; $F(1, 147) = 58.87$, $p<.001$. The effect size is large with partial $\eta^2 = .286$ indicating that 28.6% of the change in SMP scores is attributed to the PD. For RQ2, there were no significant differences by grade level taught ($p=.465$). However, there were significant differences by program; $F(2, 147) = 3.71$, $p=.027$. The effect size is small with partial $\eta^2 = .048$ indicating that only 4.8% of the variance in SMP scores can be attributed to program. This statistical difference between programs in average SMPs was noted at the post-PD observation time between only Apple (M=3.36, SD=2.11) and Cherry (M=5.37, SD=2.78) programs ($p<.01$). At pre-PD observation time, all programs performed statistically similar. On average, teachers increased 2.35 (SD=2.73) SMP indicators from pre-PD to post-PD; Cherry increased most (M=2.82, SD=2.78) followed by Blueberry (M=1.82, SD=2.45) and then Apple (M=0.95, SD=2.10).

Limitations

There are limitations to this study. Our sampling frame has limitations. Teachers from Apple cohorts volunteered to participate in the PD whereas the same is not true for Blueberry cohorts. Broadly speaking, more than half of the teachers from Blueberry cohorts were (a) required to attend by school or district-level personnel or (b) strongly encouraged by peers who decided to participate. Thus, those who are less motivated to complete long-term PD may have different outcomes making instructional changes. Moreover, many of the teachers from the Apple cohort participated in PD between 2012-2015 as part of the Cherry cohort. It is plausible that there may be a ceiling effect for average promotion of SMPs during instruction, which limited the mean growth for Apple teachers.

Importance of the Research

Taken collectively, these quantitative findings suggest that on average, teachers provided more opportunities for students to engage in the SMPs after the PD. All teachers showed growth in their promotion of SMPs after experiencing more than 100 hours of PD. There were no statistically significant differences across cohorts of teachers but there was a difference in the frequency with which elementary and secondary teachers promoted the SMPs during their instruction. These results have implications that connect research and practice. First, we noticed that instructional opportunities were clearly influenced by the implementation of teachers' choices of task, changes in learning environment, and ways discourse was promoted (see Bostic & Matney, 2014a; 2016 for discussion). Teachers internalized the standards for teaching and learning mathematics (NCTM, 2007) in ways that resonated with their instruction in the Common Core-era. Second, Guskey and Yoon's (2009) framework provided a means for us, as mathematics teacher educators, to frame our PD. This study adds convergent evidence that adhering to key features of PD leads to significant outcomes for PD participants.

In conclusion, the results broadly suggest that PD drawing upon Guskey and Yoon's key features as well as focusing on the CCSSM (CCSSI, 2010) and NCTM's Standards (2007) has potential to lead to changes in the way K-10 teachers designed and implemented mathematics instruction, as evidenced by teachers involved in this program. The SMPs do not dictate curriculum or teaching but they do provide ideas for engaging students in ways that promote mathematics proficiency during classroom instruction. PD may help mathematics teachers at all grade levels make sense of mathematics instruction that supports students' appropriate mathematical behaviors.

Endnote

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