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Developmental Perspective into Students' Contextualization of Problem Solving

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The purpose of this paper is to investigate students' contextualization of problem solving, not the problems. This study draws on the naturalistic paradigm and uses a developmental perspective to explore students' representations and metaphors used during problem solving. Students of comparable abilities employed similar representations, tended to use analogous metaphors during problem solving, and perceived solutions as outside of a problem's context.

Keywords: Linguistics, Problem solving, Reasoning and Proof

Introduction

Problem solving is central to mathematics and instruction should give students daily experiences with it (Kilpatrick, Swafford, & Findell, 2001). Prior problem-solving experiences including teachers' scaffolding or suggestive language influence students' problem-solving behaviors and perceptions (Lesh & Zawojewski, 2007). The aim of this paper is to understand how students' contextualize problem solving. We illuminate relationships between students' problem-solving performance and experiential expressions via metaphors and representations employed during problem solving.

Related Literature

Embodied Cognition

The theoretical framework for this study stems from the embodied cognition perspective (Lakoff & Núñez, 2000). Students' problem solving is influenced by the cognitive network (i.e., beliefs and academic knowledge) and external relationships with the environment and other individuals (Lesh & Zawojewski, 2007). Prior experiences are difficult to communicate at times for teachers and students, but linguistic tools, such as metaphors, used by students can be rich with representational elements (Kövecses & Benczes, 2010). Metaphors denote one figure of speech as another figure of speech (Merriam-Webster, 2011). They embody experiences and are a means to support transfer through language, thought, and action.

Problem Solving and Representations

A problem is a developmentally appropriate challenge for which a problem solver has a goal but the means for achieving it are not immediately apparent (Schoenfeld, 2011). It requires making sense of the problem and the involved decisions to achieve the desired goal (Schoenfeld, 2011). When solving a problem, the existence of "a" solution or "the" solution is uncertain. Moreover, a pathway to such solutions is unclear (Schoenfeld, 2011). Research on students' problem solving indicates that prior experiences and knowledge, beliefs and dispositions, and culture play a huge role in how individuals approach problem solving (Lesh & Zawojewski, 2007).

Representations characterize a product or process (Goldin, 2002), or more specifically "an item that corresponds in an iconic sense to another item, an 'original' to which it refers" (von Glasersfeld, 1985, p. 2). Re-presentation characterizes a "conceptual construct that has no explicit reference to something else" (von Glasersfeld ,1985, p. 2). This distinction is critically linked to a contextualized understanding of mathematics (Goldin, 2002). Learners encode familiar contexts as internal representations such as beliefs, competencies, and expectations (Goldin, 2002). These internal representations are (a) based on everyday experiences, (b) shared by many, (c) extensively linked within one's cognition, (d) developed prior to learning mathematics in a context, and (e) supported by one's culture (Goldin, 2002). Thus, prior experiences greatly impact students' perceptions (i.e., representations) of problem solving (Schoenfeld, 2011).

Metaphors

As representations associate one item to an iconic other, the linguistic, cognitive counterpart is the conceptual metaphor. Current conceptual metaphor theory includes the literal component and conceptual component (Lakoff & Johnson, 2003). The literal component is the actual literal expression, while the conceptual metaphor is a mapping between two objects: the source and the target domain. The source domain is the experientially-known domain and the related concept is the target domain. For example, "Your theoretical framework has a solid foundation" would involve the conceptual metaphor of "THEORIES ARE BUILDINGS". The target domain is theoretical framework and source domain is building. Variations of being (e.g., are and were) indicate unidirectional flow from the target to source domain. Conceptual metaphors can be classified in one of three hierarchical categories: structural, ontological, and orientational (Kövecses & Benczes, 2010; Lakoff & Johnson, 2003). Structural metaphors tend to describe a complex concept, such as time or understanding, in terms of a concrete experiential object, such as a limited resource (i.e., "DON'T WASTE MY TIME"). Ontological metaphors employ less structured target domains and necessitate a new defined reality to understand the shared experience. Personifications are regularly ontological. Orientational metaphors broadly conceptualize a specific direction inherent in human development. For example, the literal expression, "Things are looking up" demonstrates the conceptual metaphor of GOOD IS UP. Conceptual metaphors are used to map how individuals' cognitive domains are related to expression of their experiences (Lakoff & Johnson, 2003).

The relationship between the experiences of the teacher and student are vital to mathematics education. Teachers and students share an experiential set: solving mathematics problems. However, the student's and teacher's perspectives of what constitutes mathematical problems and/or solutions are complex in structure (Lakatos, 1976). Metaphors are culturally designed to articulate these implicit perspectives, and they have been found to encourage and incite cognition (Lakoff & Núñez, 2000).

Research Questions

The two research questions are: (1) How do middle and high school students' problem solving compare? (2) How do middle and high school students contextualize problem solving?

Method

Research Design

This study drew on a naturalistic paradigm and phenomenological inquiry to closely examine students' contextualization of problem solving (Short, 1991). Researchers employed a developmental perspective to explore students' problem solving.

Participants

Six participants for this qualitative study were representatively selected from investigations with larger samples. Data from sixth-, tenth-, and eleventh-grade students were collected during a think aloud conducted during two prior studies. Three middle and high school students from each study were selected. One sixth- and eleventh-grade pair (i.e., Theta and Kappa) performed above average compared to participants in the larger samples. A second pair had average performance (i.e., Beta and Lambda) and a third pair performed below average compared to peers (i.e., Gamma and Mu). Pairs two and three involved sixth- and tenth-grade students.

Data Collection

All participants completed a think aloud during a 40-minute period, which was video recorded. Sixth-grade participants completed four problems and high school participants responded to three problems. All participants were asked to solve developmentally appropriate problems using materials (e.g., manipulatives and markers) provided during the interview.

Data Analysis

Three analyses were conducted with videotapes and interview transcripts. First, students' responses were scored as correct or incorrect/no response by two mathematics educators. Correct responses had (a) solutions that answered the problem and (b) representation(s) that supported the solution. Interrater agreement (IRA) was used for the first and second analyses and calculated using r_{wg}. Second, correct responses were coded using a representation coding protocol (Lesh & Doerr, 2003). Representation categories included symbolic, pictorial, tabular, verbal, concrete model, and mixed. IRA for these analyses was ideal, $r_{wg} = 1$. The third analysis was conducted by one researcher and intended to categorize students' conceptual metaphors used during the think aloud. The three conceptual metaphors were structural, ontological, and orientational (Kövecses & Benczes, 2010; Lakoff & Johnson, 2003).

Results

Participants with comparable performance tended to use similar representations. Theta and Kappa answered more problems than peers and also employed a variety of representations. Moreover, they did not immediately implement a symbolic approach like other participants. Gamma wrestled with symbolic expressions to explore one problem. Similarly, Mu read the problem and immediately combined numbers. Beta's attention focused on manipulating a concrete approach for one task, and then tried, albeit unsuccessfully, to employ symbolic representations with other problems.

Participants' metaphor use offered insight into their contextualization of problem solving. Theta and Kappa tended to use action verbs more often than their peers. For example, Kappa used "equals" more often than Lambda and Mu, who tended to use variations of "to be". As a whole, middle school participants employed metaphors far less than their high school counterparts. Kappa, Lambda, and Mu said "got" and variations of "to be" frequently whereas high school students' language was more complex in vocabulary and grammar structure. For example, Gamma stated that he was "going in the other direction" and "getting off track". These literal metaphors align with the structural conceptual metaphor of PROBLEM SOLVING IS A JOURNEY. Concomitantly, Theta had the literal metaphor, "my mind hit a wall" indicating the same conceptual metaphor as Gamma. Less successful students said "(verb) out" more often than their peers. Lambda frequently made comments like "figure out this problem", "take him [number] out", and "draw it [representation] out". These types of ontological metaphors indicated that students perceived the solution as outside of the problem's context. Thus, problem solving, as interpreted by students, can be characterized as working from within one context and outward to another where the solution lies.

Conclusion

The aim of this study was to examine students' representations, contextualizations, and metaphors of mathematical problem solving. A common theme emerged across grade levels: effective problem solvers tended to use nonsymbolic representations and more conceptual metaphors to support their problem solving. Students' contextualization suggests that problem solving is moving towards a solution, which is not readily associated with the task's context. Kappa and others' strategies often employed symbolic representations, which divorce mathematical symbols from their context. These results aligned with Santos-Trigo's (1996) findings that students perceived symbolic representations as more appropriate than others during problem solving, and students were reticent to explore nonsymbolic representational approaches. The perception of mathematics as abstract due to its highly symbolic nature may have encouraged students to disassociate the problem's context from the problem and solution. Thus, practical considerations are necessary to enhance learners' contextualization of problem solving.

This exploration also suggested a new model to draw on students' experiences. The student-described experiences with problem solving indicated that students perceived problems ontologically as containers. Linguistically, students contextualized problem solving with the ontological conceptual metaphor of PROBLEMS ARE CONTAINERS. This result was surprisingly natural as Kövecses and Benczes (2010) argue, the experiential understanding of in and out is inherent with human existence. The ontological metaphor of container is powerful and intimately involved with our perception of the world. The container (i.e., problem) held all knowledge needed to "solve" the problem. Therefore, the action of "solving" the problem was to use the given knowledge to move one's understanding from inside to outside the container.

This research led to a transition along a developmental continuum of students' perceptions of problem solving via the compass of contextualization. The proposed model can support future investigations into enhancing students' nonsymbolic representation use during problem solving and their problem-solving outcomes.

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