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Deepening Teachers' Understandings of Mathematical and Pedagogical Connectedness

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Deepening Teachers' Understandings of Mathematical and Pedagogical Connectedness: The Walk-Across Task

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For the 45 states and 3 territories that have adopted the Common Core State Standards for Mathematics (CCSSM), there continues to be a focused effort on professional development that strives to help teachers understand the meaning and intent of the standards (Editorial Projects in Education Research Center, 2013). Developing a vision for how the CCSSM do not represent *business as usual* (CCSSI, 2010) is one of professional development providers' most pressing imperatives. As teachers work to make sense of the new standards, they need to spend time considering the ways in which the standards connect to foster the development of students' mathematical understandings (Association of Mathematics Teacher Educators, Association of State Supervisors of Mathematics, National Council of Supervisors of Mathematics, and National Council of Teachers of Mathematics, 2010).

In my own work with teachers, I have found that providing the time for teachers to consider the connections among standards allows them to see that to teach the CCSSM requires a greater understanding of mathematics and the associated pedagogical content knowledge (Herrelko, 2011). Although for many teachers there remains a disproportionate ratio between the lengths of time spent teaching traditionally and research-based reform practices, I have found that rigorous study of the CCSSM offers both veteran and novice teachers a common place to converse about the nature of teaching and learning. One task I have used to help teachers make sense of the CCSSM is the Walk-Across task (see Appendix A). Simply stated, a Walk-Across is a reorganization of the standards *across* grade level, domain, and clusters with a focus on the connections among a particular subset of mathematical ideas such as fractions or algebra. In this paper, I will describe the findings and perspectives of one group of teachers who recently completed the Walk-Across task and share my views on the implications of these findings for those who lead professional developments focused on the CCSSM.

Mathematical Connectedness

The CCSSM provides key insights into particular mathematical connections within the standards by using clusters and domains of related standards. It is noted that "standards from different clusters may sometimes be closely related, because mathematics is a connected subject" (CCSSI, 2010, pp. 5). This statement that other standards outside of designated clusters may be closely related is a significant one. It can act as a point of entry into deeper exploration of not only the standards themselves, but also of the richness of mathematics. Professional development providers can engage teachers in seeing mathematical connections beyond the indicated structure of the CCSSM as the teachers work to envision a set of sense-making experiences for their students, both within their own classrooms and across their schools and districts.

In examining the mathematics content of the CCSSM, teachers need to comprehend more than what each standard means students should know and do. To cultivate a deeper knowledge of both the mathematical and pedagogical implications of the CCSSM, teachers should be given time to work together to explore the ways in which standards connect across domains and grade levels to develop proficiency in children's mathematical thinking.

Exploring Connections *across* the CCSSM

In a recent four-day professional development workshop, 23 kindergarten through fifth grade teachers from schools in three different counties in a mid-western state were given the task of designing their own Walk-Across (K-5) for fractions. During the designing of the Walk-Across, teachers were asked to show and explain what each standard relating to fractions meant students should know and be able to do, and then explicate the way in which that knowing and doing connected to prior and subsequent standards regardless of domain or grade. Teachers were put into groups of four that spanned grades K-5: two K-2 grades teachers and two 3-5 grades teachers. Teachers worked on the assignment for approximately one hour each day following planned CCSSM professional development that expected them to think through problems involving fractions and related pedagogies that allow for the Standards for Mathematical Practice (CCSSI, 2010) to emerge.

When considering the use of a task like the Walk-Across within a professional development setting, it is important to understand that the task was not offered in isolation of other professional development activities. Its value was intertwined with the other tasks being done throughout the four-day period. These activities ranged from reading and discussing effective mathematics teaching (Herrera, Kanold, Koss, Ryan, & Speer, 2007) and its relation to promoting the expected mathematical practices for students (CSSI, 2010) to the specific content knowledge tasks meant to deepen teachers' understandings of unit fractions, operations on rational numbers, and the denseness of rational numbers. Teachers also considered the teaching of others and analyzed student thinking through the use of locally produced video cases as well as selected sections of *Connecting Mathematics Ideas* by Boaler and Humphreys (2005).

At the end of the professional development, each group of teachers submitted their Walk-Across document and a reflection journal of their own mathematical and pedagogical sense making throughout the professional development. An interpretive analysis (Hatch, 2002) was used on the Walk-Across documents and reflection journals to identify salient interpretations and verify categories of what emerged for teachers during their work on the Walk-Across task. The full version of the directions for the Walk-Across task is included in Appendix A.

Teachers Growing Awareness of Mathematics Connectedness

As teachers considered how the different standards connect to develop students' mathematical knowledge, they found themselves examining a rich network of mathematical relationships. When this realization first happened, it was not uncommon for some teachers to feel overwhelmed. One teacher described it this way:

> I know we hear all the time that mathematics is connected but until we did the Walk-Across for fractions I don't think I really understood just how connected it is. There are so many ways to draw connections and see how learning so many other mathematical ideas helps the learning of later ideas its mind boggling. The most challenging part of doing the Walk-Across was not in finding the ways the standards connected, but in just trying to decide where to stop making connections.

The realization of the connectedness of mathematics can lead to this important perturbation. If mathematics is so deeply connected how does one organize it? This is something with which all teachers of mathematics need to wrestle. I encouraged the teachers to persevere in solving the task by first explicating the connections they felt were the strongest.

Teachers were especially surprised to find a number of standards relating to fractions that were in other domains and grades. In the CCSSM, the domain for fractions starts in grade three. There is not a specified domain for fractions in grades K-2. Once teachers began their focused look for fraction ideas within the K-2 standards they quickly recognized why our professional development on fractions involved elementary teachers of all grade levels. Being able to place one's instruction within the broader perspective of what students learn was another important understanding teachers gained from the Walk-Across task. On the second day of the professional development workshop, one kindergarten teacher wrote the following in his reflection journal.

> I am excited to see how further work on the 'Walk-Across' will help me gain an understanding of how what I am teaching affects what the students will learn in the higher grades. I have enjoyed the fact that since

> yesterday, I have already begun to make more of those connections and see just how much math ideas relate to one another. I think many times, we don't think of math as being related to each other outside of addition, subtraction, multiplication, and division. It is really neat to be able to see the connections that can be made not only across grade level, but also across the domains within the Common Core that helps the students develop into proficient mathematicians.

In this reflection, the teacher described how the Walk-Across aided his emergent understanding that the relations of mathematics expand beyond the four operations and more importantly that it is exciting to see the ways in which his work with kindergarten students prepares them for future learning experiences. The notion that mathematical connectedness expanded beyond the four operations and across domains and grade levels was also made evident in the teachers Walk-Across documents as will be examined in the following section.

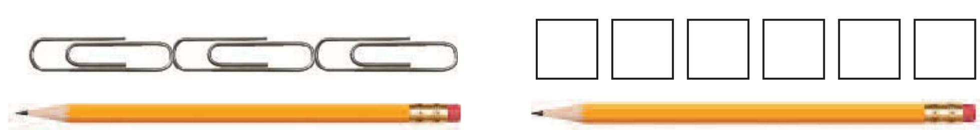
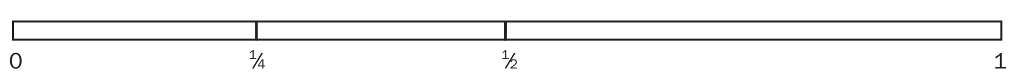
Connecting Mathematics and Learning Experiences across Domain and Grade

Teachers explicated connections to fractions from different standards found in each domain and at each grade level. Sometimes these connections were glaringly obvious, such as 1.G.3, "Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of" (CCSSI, 2010, p. 16). Other connections were made in subtle yet significant ways, showing that the teacher to develop understanding of

FIGURE 1

Walk-Across Building Fraction Understanding in the CCSS

Special Notes: This standard begins the notion of comparing fractions by realizing it requires a greater number of smaller units to measure a quantity and fewer larger sized units to measure the same quantity.

Common Core Standard	**Unpacking** What will the child know and be able to do? How does this standard connect with prior and subsequent standards regarding understanding fractions?
2.MD.2 Measure the length of an object twice, using lengths units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	The students should be given opportunities to measure objects using various length units and then compare the units used to determine how the size of the units effects final measurement of the whole object.  ***Example:*** Students will compare the measurement of a pencil using both paper clips and color tiles. They will discuss the difference in using the two different units of measure. This skill builds on 1.MD.2 in which students understand that they are measuring with non-standard units and need to be placed end to end for a measurement. This is also developed in kindergarten through K.MD.2 where students develop a sense of comparing measureable attributes and describing the difference. This is a stepping stone to 3.NF.1 and leads directly into 3.NF.2a. Students will be able to understand that a fraction is part of a whole and then represent it on a number line. ***Example:*** Students will make fraction strips by folding 5 equal strips into halves, thirds, fourths, and sixths. Using these 5 strips the students will draw tick marks to represent the fractions on the number line from zero to one.  ***Connect:*** Each paper clip is a larger unit than a color tile unit so it requires more color tiles to equal a whole object. It requires 4 fourths to equal 1 on the number line whereas it only requires 2 halves because ¼ is a smaller fractional unit than ½.

fraction ideas can utilize learning experiences in the other domains. An example of this can be seen in Figure 1 as the teachers drew upon a measurement standard from second grade to explicate its connection to later fraction ideas.

Figure 1 is an excerpt of a group of teachers Walk-Across document showing the connections they found for standard 2.MD.2. The teachers demonstrated how the measurement standards in grades 1 and 2 prepare students for fraction knowledge in grade three. They made explicit the possibility that within the educative experiences provided through the study of measurement there are opportunities for children to see an inverse relationship between the size of the unit and the number of units it will take to span the object being measured. It is in this measurement experience that a connection can be made across grade levels as students of third grade consider the inverse relationship between the size of the unit fraction 1/b and the number of unit fractions it takes to compose one whole unit.

Another set of connections explicated by teachers can be found in Figure 2 (pgs. 54-55). In this Walk-Across excerpt, the teachers focused on the third grade standard of equivalent fractions and fraction size comparison (i.e., 3.NF.3). The teachers described how student mathematical experiences and understandings connect from kindergarten through grade five. The teachers specifically detailed this by referencing the domains of counting and cardinality, measurement and data, geometry, and numbers and operations–fractions. In making these across domain and grade connections, teachers began to reconsider their own practice. Two first grade teachers explained their revelation in this way.

> As we look through the standards I can see that the building blocks for understanding division begin in Kindergarten. I appreciate the time to work through the CCSS. Time is always an issue during the school year. Taking the time to see the progression of math topics through the grades will improve my teaching... I see how important it is to know what is being taught in the other grade levels.

> Seeing what they [students] need to know and how they are being asked to show a deeper understanding in the higher grades was good for me to see. I plan to spend more meaningful time on this unit [ideas connected to fractions]. It builds on their later understanding of multiplication, division, and geometric topics to a greater degree than I ever considered.

Seeing the importance for their own understanding of the learning expected to take place across grade levels allowed teachers to describe the ways in which they wanted to change their practice. In their reflections, four categories of pedagogical change emerged. These changes were:

1. providing students time and opportunity to make their own mathematical connections;
2. providing students with worthwhile mathematics tasks to engage their intellect;
3. establishing a safe and respectful learning environment with an expectation of student sense making; and
4. becoming more adept facilitators of mathematical discourse.

As teachers worked through the Walk-Across task, they made decisions on the best way to organize their connections. On the first day of the professional development, there was much discussion on the best way to illustrate the many connections. Some groups felt that it made the most sense to start with the lower grade standards and show how later standards built upon them. In contrast, other groups started with grade three and showed how those standards were built upon by prior standards and supported notions in later standards. Still other groups started with grade five fraction standards and demonstrated how prior standards worked to build student understanding and preparedness for the fifth grade. The different ways of organizing these connections led to important discussion within the professional development about the richness and connectedness of mathematics. This occurrence supported other elements of the professional development as teachers would direct discussion in ways that:

1. went outside their own grade level considerations especially when working on mathematics tasks that were well beyond their particular teaching obligations;
2. promoted open discussion of teaching ideas with those who teach other grades;
3. encouraged mathematics task development that could be used across grade levels;
4. honestly considered the difficulties of changing one's teaching practice; and
5. challenged one another to help colleagues not participating in the professional development to understand their transforming perspective.

FIGURE 2

Walk-Across Building Fraction Understanding in the CCSS

Special Notes: Students need to be able to explain how fractions that “look” different are the same. You have to be able to think about and visualize their size. Manipulatives and numbers lines can be used to demonstrate and understand this concept. Students also have to be able to create fractions that are equal and be able to explain it to their peers using models. Whole numbers can also be expressed as fractions and this can be easily done on a number line by showing different ways to represent 1. Students also need to compare fractions with the same numerator or same denominator.

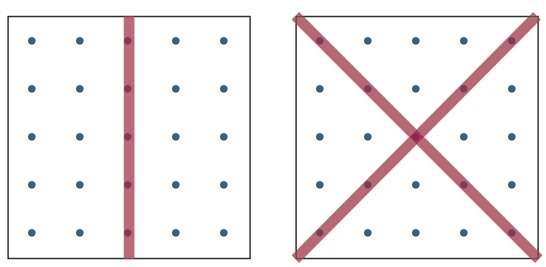
Common Core Standard	**Unpacking** What will the child know and be able to do? How does this standard connect with prior and subsequent standards regarding understanding fractions?
3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols.	The students should be able to understand and explain equivalent fractions. The will compare fractions of different sizes. They will use a number line to identify equivalency by observing that the fractions are on the same point of the number line. Recognition of equivalent fractions and generating equivalent fractions will be expected of students They will express whole numbers as fractions. Finally, they will look at fractions with the same numerator or same denominator and compare them. They will hopefully see comparisons are only true when they are using the same whole. This standard is an extension 3.NF.1 and 3.NF2. Students need to understand what each number or part of the fraction represents. For example in the fraction 1/5 the denominator represents how many parts the whole is divided into (5) and the numerator (1) represents the number of parts you are considering of the whole. Students will also have experiences seeing the relationship of fractions on a number line. See above example for the connection to a number line. Standard K.MD.2 and K.MD.3. In both of these standards the students are introduced to the academic language of compare. Students have to compare objects with measurable attributes. They also look at a specific number of objects, categorize them and count the number of each within the categories. K.CC.6 also has students grouping objects and depending on how they are grouped they are comparing if the groups are greater than, less than, and equal to each other. Standard 1.G.2 and 1.G.3 focused on students dividing shaped in to halves and fourths and describing them. They are using a variety of language for similar terms. They also see that decomposing into more equal shares makes smaller shares. ***Example:*** A fourth can also be expressed as a quarter. ***Example:*** Geo Board Activity  Standard 2.MD.2 has a great connection. Students spend time measuring objects using unit of two different lengths and they observe and describe the number of units relates to the size of the unit chosen. ***Example:*** If using cubes and large paper clip to measure the height of a bottle the students will see it takes more cubes than paper clip to measure the bottle (4 paper clip to 11 cubes). The cubes were smaller so it took more of them to measure the height verses the paper clip. This will help students to understand later in fractions that as the denominator increase the unit is smaller. It is a great visual for them to see in the earlier grades. Standard 2.G.2 and 2.G.3 have a connection. Students are asked to divide rectangles into rows and columns of same-size squares and count to find the total number. This is a good connection to equivalent fractions. Students are also asked to divide circles and rectangles into two, three,

FIGURE 2 (cont.)

<table>
<tr><th colspan="2">Walk-Across Building Fraction Understanding in the CCSS (cont.)</th></tr>
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and four equal shares and to describe them using words such as halves, thirds, half of, a third of in addition to describing the whole as two halves, etc. which also allow students to “see” equivalent fraction.
Example: Students can complete a paper-folding activity. Fold your paper in half. (How many parts is your paper divided into? How many halves make a whole?) Fold your paper in half again and open it up. (How many parts is you paper divided into now? How many fourths make a whole. How many fourths make a half?)
Standard 4.NF.1 and 4.NF.2 have connections. Students are extending their understanding of equivalence. They have to explain why fractions are equivalent. Students must explain how and why they add and subtract fractions and compare fractions with the same denominator. Also, they need to be able to find and tell why they use equivalent fraction for adding and subtracting fractions and how to compare fractions with unlike denominators.
Example: Fraction bars, fraction circles, and a number line are examples of materials that can be used to explain how to add, subtract, and compare fractions with both like and unlike denominators.


The standards 5.NF.1 and 5.NF.2 are extensions of 3.NF.3. At fifth grade students use equivalent fractions as a strategy to add and subtract fractions. Students are expected to develop fluency with adding, subtracting, and comparing fractions and mixed numbers with the same or different-denominators. Mixed numbers is an addition from fourth grade. Students are also solving word problems involving the addition and subtraction of fractions. Students are also using estimation skills to assess if students answers are reasonable.
Example: 2/3 + 5/4 = 8/12 + 15/12 =23/12 or 2/5 + 1/2 = 3/7 (knowing 3/7 less than 1/2)
</td></tr>
<tr><td>Resource References:</td><td>Geo board activity, taken from Dr. G. Matney, Summer 2012, CORES Elementary ITQ Grant
Equivalent fraction image, taken from http://aschouten.wordpress.com
Fraction circle image, taken from http.//hr6math.com
Sample addition of fraction problem, taken from the Common Core State Standards for Mathematics (2010)</td></tr>
</table>

In directing their own discussion in these ways, it became apparent that by creating their own Walk-Across for fractions teachers were generating self-selected questions for discussion. Examples of such questions include: What does the connectedness of mathematics mean for teaching and learning? How do I determine an appropriate entry point into a mathematics topic or does it even matter since it is all connected? When giving a rich task in which students begin to see connections that I have not considered or cannot make sense of, how do I respond as the teacher? These questions demonstrate that the Walk-Across task was supporting teachers’ pedagogical considerations as they began to see their own need for greater mathematics understanding and stronger pedagogical content knowledge.

Teachers elected to find and use many resources beyond the documents produced by their own state. As they did this, they began to see patterns in what was being said across states and felt as though seeing the same thing said in different ways and with different examples was helpful; not only in their unpacking of the meaning of each standard, but also in seeing more clearly the mathematical

Table 1: Areas of Emerging Confidence Related to the Walk-Across Task

Area of Confidence	Sample Text from Teacher
Personal Mathematics Knowledge and Ability	In my work with the Walk-Across I have learned a lot of mathematics. I did not think at first looking closely at the standards would teach me anything, but boy was I wrong. Learning about the division problem types and how understanding division connects with fractions are a couple examples. I have grown mathematically in what I know and I can (and HAVE!) solve problems involving fractions that I previously never made sense of.
Personal Pedagogical Knowledge to Help Students Understand Mathematics	As I was struggling to come up with an example for a fifth grade example today, I realized I had done a lesson from NCTM's Navigation Algebra book that fit that standard completely. It would have taken me a long time to make this connection had I not done the Walk-Across. Our completed Fraction Walk-Across turned-out well. It was a lot of work but I feel more confident in planning my teaching since I can see how everything fits together.
Knowledge to Help other Teachers Understand the CCSSM	I really enjoyed seeing some light at the end of the tunnel so to speak when we got into the Walk-across discussion at the end of the day. I feel that it is beneficial to look at the Common Core in this way because you can visually see the connection. I feel that with what we are creating with these Walk-Across documents, we will be able to express to our districts what is to be going on with the Common Core.

connections between standards at different grade levels and domains. One teacher's reflection reveals this.

> After looking at the Ohio Model Curriculum we read what Utah and North Carolina had as well. Then the AH HA moment came and we started seeing how the Standards for Mathematics Practice connected to how students should be interacting with each content standard through representing their own thinking via drawings, patterns, and manipulatives.

Through the development of their Walk-Across, each group of teachers found different resources to rely on and shared their discoveries with others. The finding and sharing of additional resources to complete the Walk-Across task provided the opportunities for teachers to begin to build a learning community and beneficially incorporate the aspects of the learning community throughout the other parts of the professional development.

Emerging Confidence

Through their reflections, teachers expressed a growing sense of confidence, in part due to the Walk-Across task and also from their solving of mathematics tasks above and below their grade levels. They articulated a growing confidence in one of three areas. Table 1 gives the three areas and a representative example from teachers' reflections.

Providing ways to authentically enable teachers to find confidence in their study of mathematics, pedagogical practice, and leading the way for others in their districts is certainly one of the challenges for any professional development. The emergence of these forms of confidence came through the teachers' hard work. Several of them mentioned in their reflections that they spent time outside of the professional development hours working on the Walk-Across and that in the beginning they were "a little overwhelmed." After all, the work of a Walk-Across is not easy. As is exemplified in the final reflections above, by the end of the fourth day the teachers came to value the rigorous thinking they did about how the standards connected.

Implications for Leadership Practice

Beyond the analysis of teachers' reflections, there were several noteworthy elements brought into the other parts of the professional development that may not have occurred without the teachers work in creating the Walk-Across. For example, throughout the professional development, I asked the teachers to describe any relevance they found in the mathematical tasks we were doing in comparison to their Walk-Across explorations and creations. As the teachers realized that I was not going to provide these connections, they began to share their own ideas about the tasks we were doing in the rest of the professional development and the specific standards they were exploring in the Walk-Across. Furthermore, teachers often recognized that the mathematics tasks of which we were making sense dealt with standards that they had not previously considered as being connected to fractions. This reciprocal interplay happened each day of the professional development.

The findings of this study align with Hsu, Kysh, Resek, and Ramage's (2012) work to change teachers' conceptions of mathematics. In their study, Hsu and colleagues demonstrated the importance of the interplay between transforming one's teaching practice and one's conception of mathematics. Furthermore, they described why it is problematic to just tell teachers with "charisma and authority" (p. 38) what to do in their classrooms. When thinking about the experiences we provide for teachers, we need to be careful with what it is we tell them. We should be careful not to substitute our authority for their reason any more than we would ask them to use their authority to cajole a student into the teacher's way of understanding mathematical ideas. The teachers in this study reflected that their prior learning experiences with mathematics led them to develop a conception of the discipline as one of disconnected bits of knowledge. Interestingly, teaching itself, with its daily curriculum maps, bells, and other such delineations holds an impinging logic that teaching is also done in discrete bits and pieces. Professional developments should pursue the difficult task of providing opportunities for teachers to understand mathematics connectivity and its relationship to pedagogies that promote student sense making. The Walk-Across task is appears to hold potential benefit toward meeting this goal.

The nature of mathematical connectedness alone is not enough to ensure students experience the learning of mathematics in connected and meaningful ways. The teacher is a vital interlocutor in the student's discourse with mathematics, whose own understandings and perceptions must work to facilitate meaningful and connected experiences for students' mathematics learning. For this reason, it is important to provide opportunities for teachers to make their own associations among the CCSSM and other parts of professional development devoted to their exploration of things such as worthwhile tasks, classroom norms, discourse, and the nature of mathematics. Through their work in creating their own Walk-Across for fractions, the teachers in this professional development exhibited more connected ways of seeing mathematics and their teaching practice. Through the Walk-Across discussions, the teachers became more open to their development of adaptiveness to students' mathematical ideas and emergent teaching scenarios (Ball & Bass, 2000; Ma, 1999).

Conclusion

When one begins to see mathematics as multiplicity of connections rather than a single linearity of discrete domains, it challenges the notion that good mathematics instruction is done through the learning of minutely focused bits of process and formal representation disseminated from the teacher to the student. Affording teachers of mathematics with the opportunities and support for seeing mathematics as connected ideas across the CCSSM domains and grade levels provides occasions of pedagogical awareness for the teachers to re-organize curricular experiences that allow for student sense making.

References

Association of Mathematics Teacher Educators, Association of State Supervisors of Mathematics, National Council of Supervisors of Mathematics, & National Council of Teachers of Mathematics. (2010). *Report of the joint task force on Common Core State Standards.* Retrieved from NCTM website: http://www.nctm.org/uploadedFiles/About_NCTM/President/Messages/Shaughnessy/2010_1104_PresMess_B.pdf

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning.* Portsmouth, NH: Heinemann.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: The National Governors Association for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/the-standards.

Editorial Projects in Education Research Center. (2013). *Findings from a national survey of teacher perspectives on the Common Core.* Retrieved from http://www.edweek.org/media/epe_survey_teacher_perspctives_common_core_2013.pdf

Hatch, A. (2002). *Doing qualitative research in education settings.* Albany, NY: State University of New York Press.

Herrelko, J. M. (2011). Reflections on creating strong mathematics coaching programs. *NCSM Journal of Mathematics Education Leadership, 13*(2), 39-43.

Herrera, T., Kanold, T. D., Koss, R. K., Ryan, P., & Speer, W. R. (2007). Standards for Teaching and Learning Mathematics. In T. S. Martin (Ed.), *Mathematics teaching today* (pp. 15-60). Reston, VA: National Council of Teacher of Mathematics.

Hsu, E., Kysh, J., Resek, D., & Ramage, K. (2012). Changing teachers' conception of mathematics. *NCSM Journal of Mathematics Education Leadership, 14*(1), 37-52.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Erlbaum.

APPENDIX A

Develop a K-5 Walk-Across for CCSSM Fractions

You, along with your across grade level team of 3 or 4, will create a Walk-Across document that contains the K-5 CCSSM related to the learning of fractions and shows your understanding of two important aspects of the CCSSM:

1) What does each standard mean a student should know and be able to do?

2) How does this standard connect with prior and subsequent standards regarding understanding fractions?

You should identify each standard that works to build students' understandings of fractions and concisely show with pictures, graphics, and text what the child should know and be able to do. There should also be a well explicated connection to any related prior standards and related subsequent standards.

To begin, you should give attention to each standard, regardless of domain, and consider whether or not it pertains to one's understanding of fractions. You should only include standards that you feel DO pertain to fractions. Next, consider how each standard that pertains to ideas involving fractions should be accompanied by an explanation of how prior standards prepared students for this standard, and how this standard prepares students for future standards. While explaining the prior and subsequent standards connection, only the name should be listed (for example 3.G.2) and not the full text.